



جامعة نجران  
كلية الهندسة  
قسم الهندسة الكهربائية

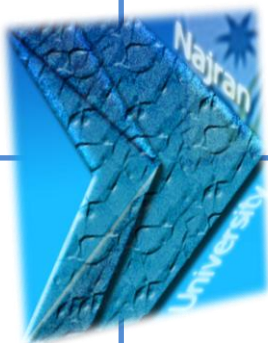
# Lecture notes And Exercises On

# STATICS

By Dr Abdulwahab Amrani

GE 101

101 هعم



## Course Objectives

To understand and use the general ideas of force vectors and equilibrium of particle and rigid body.

To understand and use the general ideas of structural analysis and internal force and friction.

To understand and use the general ideas of center of gravity, centroids and moments of inertia.

### أهداف المقرر

تزويد الطالب بالمفاهيم العامة حول:

- كيفية تحليل القوى و حل معادلات اتزان الأجسام.
- التحليل الإنشائي و القوى الداخلية و الاحتكاك.
- كيفية حساب المركز الهندسي و مركز الثقل و عزم القصور.



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# Chapter

One

## General Principals

قال النبي صلى الله عليه وسلم:

تعلموا العلم، فان تعلمه الله خشية،  
وطلبه عبادة،

ومذاكرته تسبيح،

والبحث عنه جهاد،

و تعليمه لمن لا يعلمه صدقة

وبذله لأهله قربة.

# General Principals

## 1.1 Introduction

The subject of statics developed very early in history because its principles can be formulated simply from measurements of geometry and force. **Statics is the study of bodies that are at rest or move with constant velocity.** We can consider statics as a special case of **dynamics**, in which the **acceleration is zero**.

يهتم علم الإستاتيكا بدراسة و تحليل  
الأجسام التي هي في حالة السكون

"كمية الطول": هو تعبير لقياس شيء ما  
ويستعمل لتحديد أماكن النقاط.

## 1.2 Fundamental Concepts

Before we begin our study, it is important to understand the meaning of certain fundamental concepts and principles.

دور "كمية الزمن" مهمة جدا في علم  
الديناميكا.

**Length:** Length is used to **locate the position** of a point in space and thereby describe the size of a physical system.

الكتلة عبارة عن خاصية فيزيائية للأجسام،  
وتعرف على إنها مقدار ما يحويه الجسم من  
مادة.

**Time:** Although the principles of statics are **time independent**. This quantity plays an **important** role in the study of **dynamics**.

القوى هي كميات متجهة لها مقدار و اتجاه  
ونقطة تأثير.

**Mass:** Mass is a **measure of a quantity of matter**.

الجسيم له كتلة و ليس له أي حجم.

**Force:** Force is considered as a "**push**" or "**pull**" exerted by one body on another. This interaction can occur when there is direct contact between the bodies, such as a person pushing on a wall. A force is completely characterized by its **magnitude, direction, and point of application**.

الجسم الجاسي هو الحالة المثالية لجسم صلب  
المسافة بين الجسيمات تبقى فيه ثابتة عبر  
الزمن بغض النظر عن القوى الخارجية المطبقة  
عليه. عادة ما يعتبر الجسم الجاسي على أنه  
توزيع مستمر للكتلة.

**Particle:** Particle has a **mass**, but its **size can be neglected**.

**Rigid Body:** A rigid body can be considered as a **combination** of a **large number of Particles**.

يظل الجسم في حالته الساكنة (إما السكون  
النم أو التحريك في خط مستقيم بسرعة  
ثابتة) ما لم تؤثر عليه قوة تغير من هذا الحالة.  
يشير هذا القانون أنه إذا كان مجموع  
الكميات الموجهة من القوى التي تؤثر على  
جسم ما صفرا، فسوف يظل هذا الجسم  
ساكنا. وبالمثل فإن أي جسم متحرك سيظل  
على حركته بسرعة ثابتة في حالة عدم وجود  
أية قوى تؤثر عليه مثل قوى الاحتكاك.

**Newton's first law:** A **particle originally at rest or moving** in a straight line with **constant velocity, tends to remain** in this **State** provided the particle is not subjected to an unbalanced force (Fig.1-1).

$$\sum_{i=1}^N F_i = 0$$

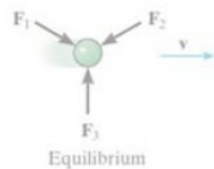


Fig. 1-1

إذا أثرت قوة أو مجموعة قوى  $(\sum F)$  على جسم ما فإنها تكسبه تسارعاً (أو عجلة)  $a$ ، يتناسب مع محصلة القوى المؤثرة، ومعامل التناسب هو الكتلة  $m$  للجسم.

**Newton's second law:** A **particle acted** upon by an unbalanced **force "F"** experiences an **acceleration "a"** that has the same direction as the **force** and a **magnitude** that is directly **proportional** to the force ( Fig. 1-2). If "F" is applied to a particle or mass "m", this law may be expressed mathematically as:

$$F = m \cdot a$$



Fig. 1-2

لكل قوة فعل قوة رد فعل تساويها في المقدار و معاكسة لها في الاتجاه.

**Newton's third Law:** The **mutual forces of action** between two particles are **equal, opposite, and collinear** (Fig. 1-3).

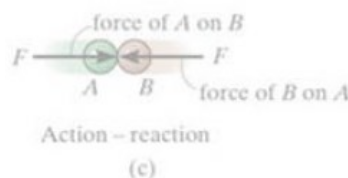


Fig. 1-3

قانون الجذب العام هو قانون فيزيائي استنباطي ينص على أنه "توجد قوة تجاذب بين أي جسمين في الكون، تتناسب طردياً مع حاصل ضرب كتلتيهما، وعكسياً مع مربع المسافة بينهما".

**Newton's Law of Gravitational Attraction:** Shortly after formulating his three laws of motion. Newton postulated a law governing the **gravitational attraction between any two particles**. Stated mathematically.

$$F = G \frac{m_1 m_2}{r^2}$$

Where

**F:** **Force of gravitational** between the two particles.

**G:** **Universal constant of gravitation**, according to experimental evidence.

$$G = 66.73 \cdot 10^{-12} \frac{\text{m}^3}{\text{kg s}^2}$$

$m_1, m_2$ : mass of each of the two particles.  
 $r$ : distance between the two particles.

**Weight**: Weight refers to the **gravitational attraction** of the **earth** on a body or quantity of mass. The weight of a particle having a mass is stated mathematically.

$$W = mg$$

الوزن هو قوة جذب الأرض للجسم

Measurements give  $g = 9.8066 \frac{\text{m}}{\text{s}^2}$

Therefore, a body of **mass 1 kg** has a **weight of 9.81 N**, a 2 kg body weights 19.62 N, and so on (Fig. 1-4).

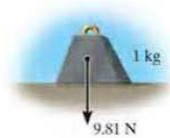


Fig. 1-4

## Units of Measurement:

- **SI units**: The **international System of units**. Abbreviated SI is a **modern version** which has received worldwide recognition. As shown in Tab 1.1. The SI system defines **length in meters (m)**, **time in seconds (s)**, and **mass in kilograms (kg)**. In the SI system the unit of force, the **Newton** is a **derived unit**. Thus, 1 Newton (N) is equal to a force required to give 1 kilogram of mass and acceleration of  $1 \frac{\text{m}}{\text{s}^2}$ .
- **US customary**: In the **U.S. Customary** system of units (**FPS**) **length** is measured in **feet (ft)**, **time** in **seconds (s)**, and **force** in **pounds (lb)**. The unit of **mass**, called a **slug**, **1 slug** is equal to the amount of **matter** accelerated at  $1 \frac{\text{ft}}{\text{s}^2}$  when acted upon by a **force of 1 lb** ( $1 \text{ slug} = 1 \frac{\text{lb s}^2}{\text{ft}}$ ).

النظام الدولي للوحدات هو نظام وحدات القياس الأوسع انتشاراً في العالم، وهو يستخدم في أغلب دول العالم. يسمى هذا النظام بالنظام المتري.

و هو نظام وحدات قياس يستخدم بكثرة في الولايات المتحدة الأمريكية و بريطانيا.

**Table 1.1 Systems of Units**

Name	Length	Time	Mass	Force
International Systems of Units	<i>meter</i>	<i>seconds</i>	<i>kilogram</i>	<i>Newton*</i>
<b>SI</b>	<i>m</i>	<i>s</i>	<i>kg</i>	<i>N <math>\frac{kg \cdot m}{s^2}</math></i>
US Customary	<i>foot</i>	<i>second</i>	<i>Slug*</i>	<i>pound</i>
<b>FPS</b>	<i>ft</i>	<i>s</i>	<i><math>\frac{lb \cdot s^2}{ft}</math></i>	<i>lb</i>
*Derived unit				

**Conversion of Units:**

Table 1.2 provides a set of direct conversion factors between **FPS** and **SI** units for the basic quantities. Also in the FPS system, recall that:

$$1 \text{ ft} = 12 \text{ in (inches)}$$

$$1 \text{ mile} = 5280 \text{ ft}$$

$$1 \text{ kp (kilo pound)} = 1000 \text{ lb}$$

$$1 \text{ ton} = 2000 \text{ lb}$$

**Table 1.2 Conversion factors**

Quantities	Unit of Measurement (FPS)	equals	Unit of Measurement (SI)
<b>Force</b>	lb		4.448 N
<b>Mass</b>	slug		14.59 kg
<b>Length</b>	ft		0.3048 m

**Prefixes:** When a **numerical quantity** is either very **Large** or very **small**, the units used to define its size may be modified by using a **prefix**. Some of the prefixes used in the SI system are shown in Table 1.3. Each represents a **multiple** or **submultiples** of a unit which, if applied successively, moves the decimal point of a numerical quantity to every third place. For example, 4000000N=4000kN (kilo-newton)=4MN (mega-newton), or 0.005m=5mm (milli-meter).

البادئة

(multiple) المضاعفات الأعلى  
(submultiple) المضاعفات الأوطاء

**Table 1.3 Prefixes**

	Exponential Form	Prefix	SI Symbol
<b>Multiple</b>			
1 000 000 000	$10^9$	giga	G
1 000 000	$10^6$	mega	M
1 000	$10^3$	kilo	K
<b>Submultiple</b>			
0.001	$10^{-3}$	milli	m
0.000 001	$10^{-6}$	micro	$\mu$
0.000 000 001	$10^{-9}$	nano	n



**Exercise 1.1:**

Convert  $2 \frac{\text{km}}{\text{h}}$  to  $\frac{\text{m}}{\text{s}}$ . How many  $\frac{\text{ft}}{\text{s}}$  is this?

$$\text{Ans: } 2 \frac{\text{km}}{\text{h}} = 0.556 \frac{\text{m}}{\text{s}} = 1.82 \frac{\text{ft}}{\text{s}}$$

**Exercise 1.2:**

Convert the quantities  $300 \text{ lb} \cdot \text{s}$  and  $52 \frac{\text{slug}}{\text{ft}^3}$  to appropriate SI units.

$$\text{Ans: } 300 \text{ lb} \cdot \text{s} = 1.33 \text{ kN} \cdot \text{s} \quad 52 \frac{\text{slug}}{\text{ft}^3} = 26.8 \frac{\text{Mg}}{\text{m}^3}$$

**Exercise 1.3:**

Evaluate each of the following and express with SI units having an appropriate prefix:

(a)  $(50 \text{ mN})(6 \text{ GN})$       (b)  $(400 \text{ mm})(0.6 \text{ MN})^2$       (c)  $\frac{45 \text{ MN}^3}{900 \text{ Gg}}$

$$\text{Ans: } (50 \text{ mN})(6 \text{ GN}) = 300 \text{ kN}^2 \quad (400 \text{ mm})(0.6 \text{ MN})^2 = 144 \text{ Gm} \cdot \text{N}^2 \quad 45 \frac{\text{MN}^3}{900 \text{ Gg}} = 50 \frac{\text{kN}^3}{\text{kg}}$$

**Exercise 1.4:**

Round off the following numbers to three significant figures:

(a)  $4.65735 \text{ m}$       (b)  $55.578 \text{ s}$       (c)  $4555 \text{ N}$       (d)  $2768 \text{ kg}$   
 Ans: (a)  $4.66 \text{ m}$       (b)  $55.6 \text{ s}$       (c)  $4.56 \text{ kN}$       (d)  $2.77 \text{ Mg}$

**Exercise 1.5:**

Represent each of the following combinations of units in the correct SI form using an appropriate prefix:

(a)  $\mu\text{MN}$       (b)  $\text{N}/\mu\text{m}$       (c)  $\text{MN}/\text{ks}^2$       (d)  $\text{kN}/\text{ms}$ .  
 Ans: (a)  $\text{N}$       (b)  $\frac{\text{MN}}{\text{m}}$       (c)  $\frac{\text{N}}{\text{s}^2}$       (d)  $\frac{\text{MN}}{\text{s}}$

**Exercise 1.6:**

Represent each of the following combinations of units in the correct SI form:

(a)  $\text{Mg}/\text{ms}$       (b)  $\text{N}/\text{mm}$       (c)  $\text{mN}/(\text{kg} \cdot \mu\text{s})$ .  
 Ans: (a)  $\frac{\text{Mg}}{\text{ms}} = \frac{\text{Gg}}{\text{s}}$       (b)  $\frac{\text{N}}{\text{mm}} = \frac{\text{kN}}{\text{m}}$       (c)  $\frac{\text{mN}}{\text{kg} \cdot \mu\text{s}} = \frac{\text{kN}}{\text{kg} \cdot \text{s}}$

**Exercise 1.7:**

A rocket has a mass of  $250 \cdot 10^3$  slugs on earth. Specify (a) its mass in SI units and (b) its weight in SI units. If the rocket is on the moon, where the acceleration due to gravity is  $g_m = 5.30 \text{ ft/s}^2$ , determine to 3 significant figures (c) its weight in units, and (d) its mass in SI units.

$$\text{Ans: } (a) 3.65 \text{ Gg} \quad (b) W_e = 35.8 \text{ MN} \quad (c) W_m = 5.89 \text{ MN} \quad m_m = m_e = 3.65 \text{ Gg}$$

**Exercise 1.8:**

If a car is traveling at  $55 \text{ mi/h}$ , determine its speed in kilometers per hour and meters per second.

$$\text{Ans: } (a) 88.514 \frac{\text{km}}{\text{h}} \quad (b) 24.6 \frac{\text{m}}{\text{s}}$$

**Exercise 1.9:**

The Pascal (Pa) is actually a very small units of pressure. To show this, convert  $1 \text{ Pa} = 1 \text{ N/m}^2$  to  $\text{lb/ft}^2$ . Atmospheric pressure at sea level is  $14.7 \text{ lb/in}^2$ . How many Pascals is this?

$$\text{Ans: } (a) 1 \text{ Pa} = 20.9 \cdot 10^{-3} \frac{\text{lb}}{\text{ft}^2} \quad (b) 1 \text{ ATM} = 101.34 \text{ kPa}$$

**Exercise 1.10:**

Two particles have a mass of 8 kg and 12 kg, respectively. If they are 800 mm apart, determine the force of gravity acting between them. Compare this result with the weight of each particle.

Ans: (a)  $F = 10.0 \text{ nN}$

**Exercise 1.11:**

Determines the mass in kilograms of an object that has a weight of:

(a) 20 mN

(b) 150 kN

(c) 60 MN

Ans: (a)  $m = 2.04 \text{ g}$

(b)  $m = 15.3 \text{ Mg}$

(c)  $m = 6.12 \text{ Gg}$

# Chapter

Two

## Force Vectors

تعلم فليس المرء يولد عالماً      وليس أخو علم كمن هو جاهل  
وإن كبير القوم لا علم عنده      صغير إذا التفت عليه المخافل

## Force Vectors

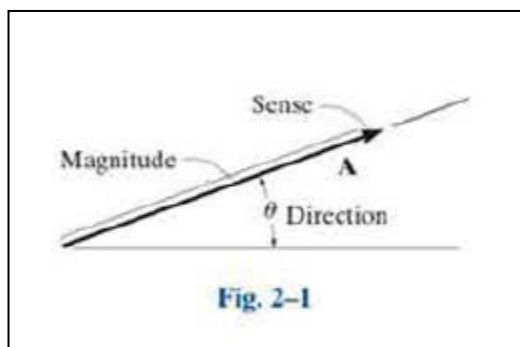
### 2.1 Scalar and vectors

A **scalar** is any **positive** or **negative** **physical quantity** that can be completely specified by its **magnitude**.

الكمية القياسية: تلك الكمية التي يمكن تعيينها بمعرفة مقدارها و وحدة قياسها.

A **vector** is any physical quantity that requires both a **magnitude** and **direction** for its complete description. A vector is shown **graphically** by an **arrow**. The **length** of the arrow represents the **magnitude** of the vector, and a fixed axis defines the **direction** of its line of action. The **head** of the arrow indicates the **sense of direction of the vector** (Fig 2-1).

الكميات المتجهة هي كميات فزيائية يمكن تعيينها بمعرفة مقدارها العددي و اتجاهها و نقطة تأثيرها و محور عملها.



For **handwritten** work, it is often **convenient** to denote a vector quantity by simply drawing an arrow on top of  $\vec{A}$ .

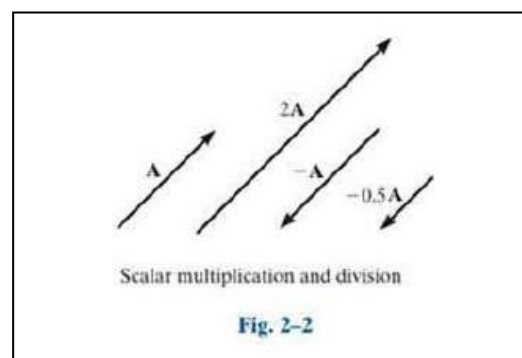
In **print**, vector quantities are represented by **bold** face letters such as  $\mathbf{A}$ , and its **magnitude** of the vector is **italicized**,  $A$ .

ضرب المتجهات

### 2.2 Vector operations

**Multiplication and division of vector by a scalar:**

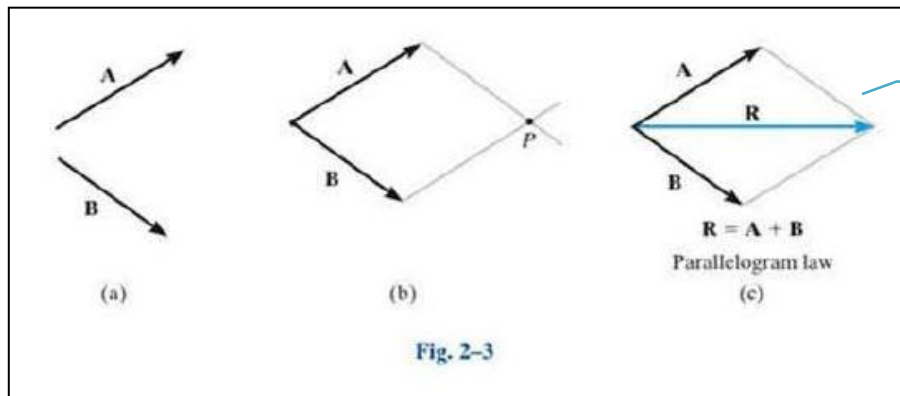
If a vector is **multiplied** by a **positive scalar**, its **magnitude is increased** by that amount. When **multiplied by a negative scalar** it will also **change the directional sense** of the vector (Fig 2-2).



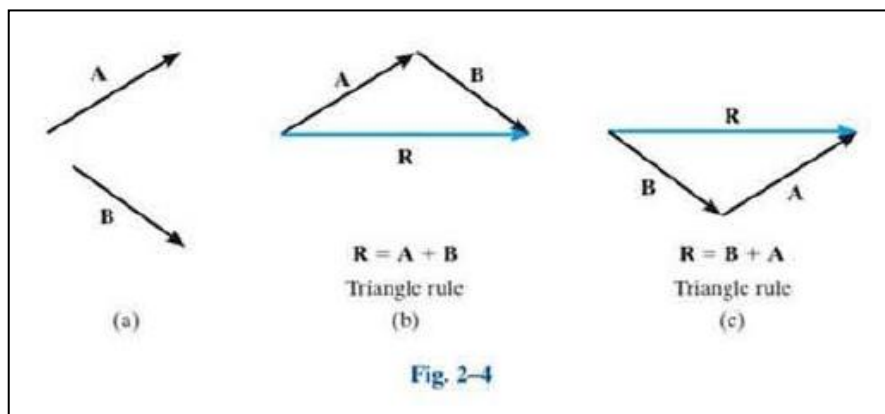
**Vector addition:**

All vector quantities **obey** the **parallelogram law** of addition. Fig 2-3 and Fig 2-4 and Fig 2-5 illustrates addition of vectors  $\vec{A}$  and  $\vec{B}$  to obtain a resultant  $\vec{R}$ .

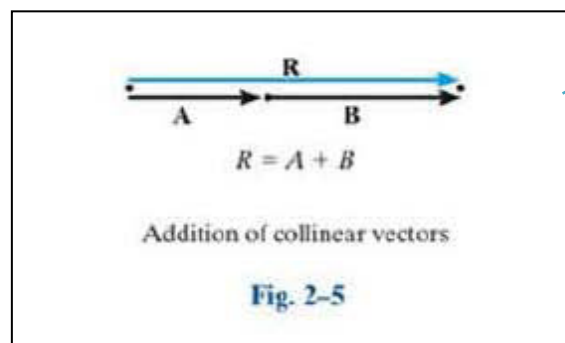
جمع المتجهات بطريقة الرسم  
البياني



قاعدة المتوازي  
الأضلاع



قاعدة المثلث



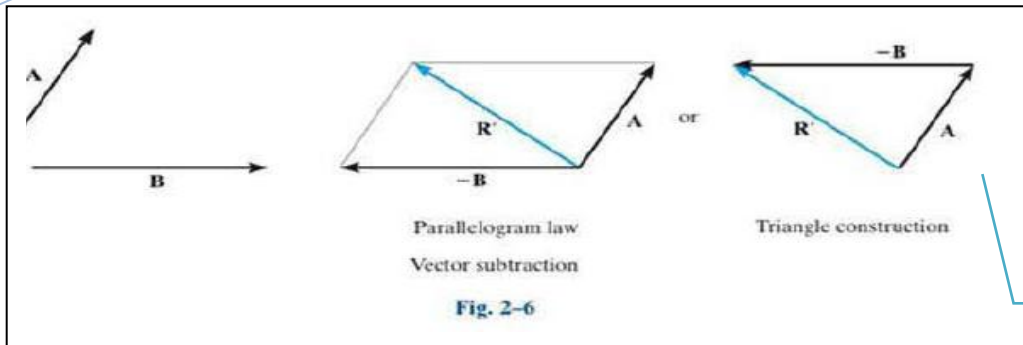
متجهات على  
نفس الخط

**Vector subtraction:**

The resultant of the difference between two vectors  $\vec{A}$  and  $\vec{B}$  of the same type may be expressed as:

$$\mathbf{R'} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

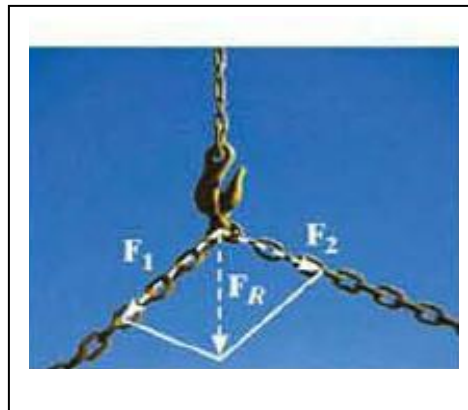
Fig 2-6 illustrates subtraction of vectors  $\mathbf{A}$  and  $\mathbf{B}$



تغيير اتجاه  
المتجه  $B$

### 2.3 vector addition of forces:

Experimental evidence has shown that a **force** is a **vector quantity** since it has a specified **magnitude**, **direction**, and **sense** and it **adds according to the parallelogram law**.



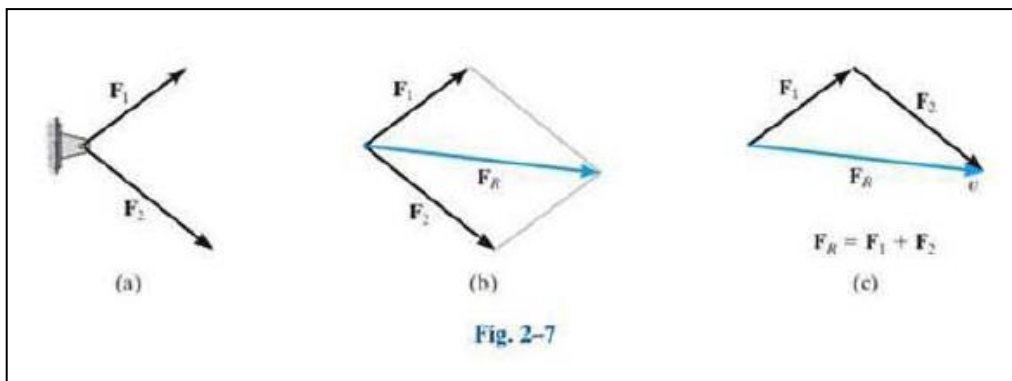
القوى عبارة عن  
متجهات

تأثير قوتين على دبوس

### Finding a resultant force:

The two component forces  $\vec{F}_1$  and  $\vec{F}_2$  acting on the pin in Fig 2-7 can be added together to form the **resultant** force

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2$$

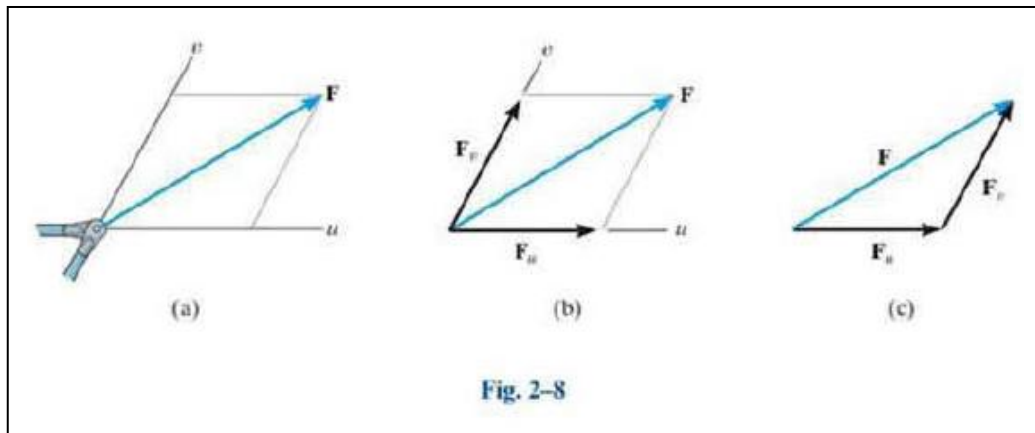


### Finding the components of a force:

Sometimes it is necessary to **resolve a force** into two components in order to **study its pulling and pushing effect** in two specific directions.

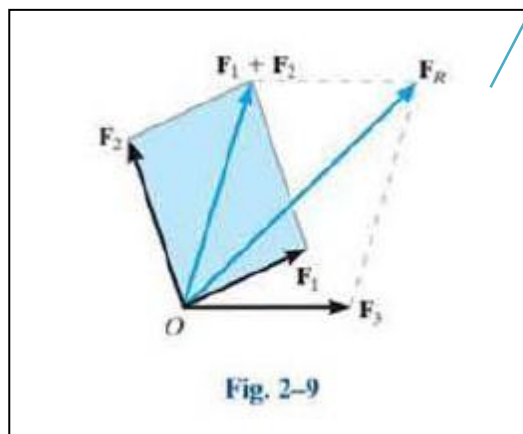
ضرورة دراسة تأثير  
القوة في اتجاهين مختلفين  
يتطلب تحويلها إلى  
مركبات

For example, in Fig 2.8,  $F$  is to be resolved into two components along two members, defined by  $u$  and  $v$  (Fig 2.8)



### Addition of several forces:

If **more than two forces** are to be **added successive applications of the parallelogram law** can be carried out in order to obtain the resultant force. For example if the three forces  $\vec{F}_1$ ,  $\vec{F}_2$ ,  $\vec{F}_3$  act at a point  $O$ , the **resultant** of any two of the forces is found ( $\vec{F}_1 + \vec{F}_2$ ) and then this resultant is **added** to the **third force** yielding the **resultant of all three forces** ( $\vec{F}_R = (\vec{F}_1 + \vec{F}_2) + \vec{F}_3$ ) (Fig 2-9).



استعمال طريقة الرسم  
البياني بشكل متتالي

هنا نجمع محصلة  $\vec{F}_1$  و  $\vec{F}_2$  مع  
المتجه  $\vec{F}_3$  للحصول على  
المحصلة الكلية  $\vec{F}_R$

### Trigonometry analysis:

**Redraw** a half portion of the parallelogram to illustrate the **triangular** head to tail addition of the components. From this triangle, the **magnitude of the resultant force** can be determined using **the law of cosines**, and its direction is determined from the **law of sines**.

The magnitudes of two force components are determined from the law of sines. The formulas are given in Fig 2-10  
cosine law:

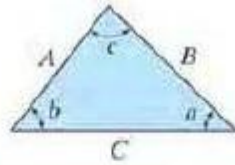
التحليل المثلثي

**sine law:**

$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

الطريقة التحليلية تستلزم قاعدة  
الجيب التمام وقاعدة الجيب  
للمثلثات

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$



Cosine law:  

$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$
 Sine law:  

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

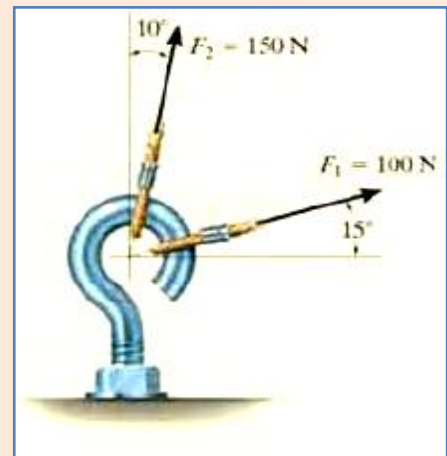
(c)

Fig 2-10

**Exercise 2.1:**

The screw eye in Fig 2-11 is subjected to two forces,  $\vec{F}_1$  and  $\vec{F}_2$ . Determine the magnitude and direction of the resultant force.

Fig 2-11

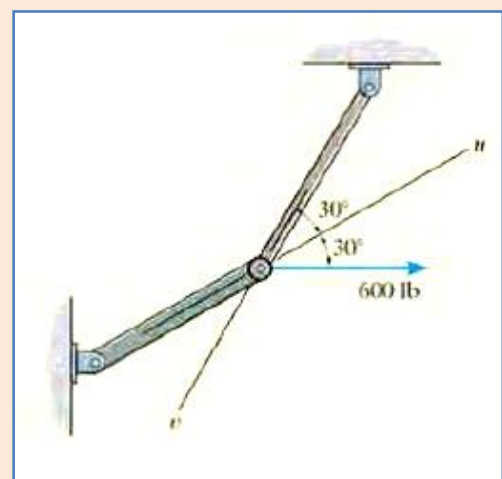


**Ans:**  $F_R = 213 \text{ N}$        $\phi = 54.7^\circ$

**Exercise 2.2:**

Resolve the horizontal 600lb force in fig 2.12 into components action along the  $u$  and  $v$  axes and determine the magnitudes of these components.

Fig 2-12



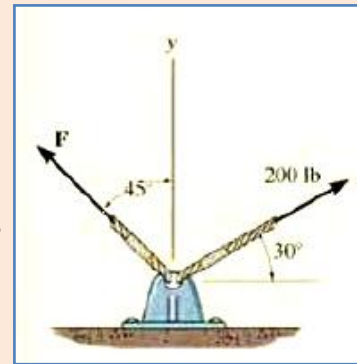
**Ans:**  $F_u = 1039 \text{ lb}$        $F_v = 600 \text{ lb}$



**Exercise 2.3:**

Determine the magnitude of the component force  $\vec{F}$  in Fig 2-13 and the magnitude of the resultant force  $\vec{F}_R$  if  $\vec{F}_R$  is directed along the positive y axis.

Fig 2-13

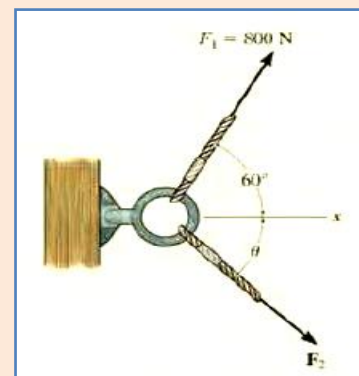


Ans:  $F = 245 \text{ lb}$       $F_R = 273 \text{ lb}$

**Exercise 2.4:**

It is required that the resultant force acting on the eyebolt in Fig 2.14 be directed along the positive x axis and that  $\vec{F}_2$  have a minimum magnitude. Determine this magnitude, the angle  $\theta$ , and the corresponding resultant force.

Fig 2-14

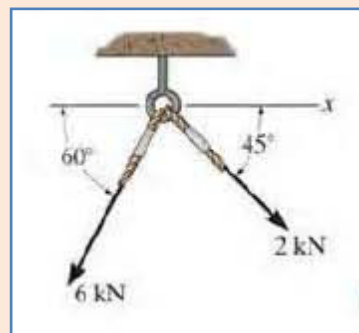


Ans:  $\theta = 90^\circ$       $F_R = 400 \text{ N}$       $F_2 = 693 \text{ N}$

**Exercise 2.5:**

Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.

Fig 2-15

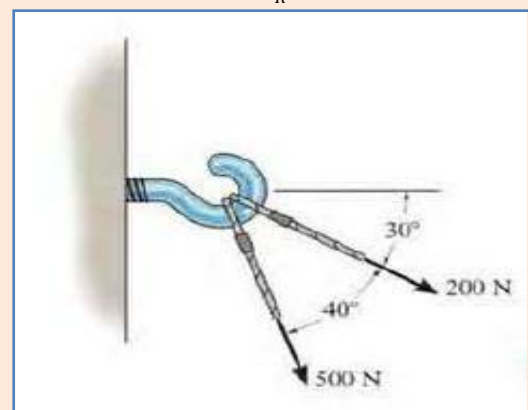


Ans:  $F_R = 6.80 \text{ kN}$       $\theta = 103^\circ$

**Exercise 2.6:**

Two forces act on the hook. Determine the magnitude of the resultant force.

Fig 2-16

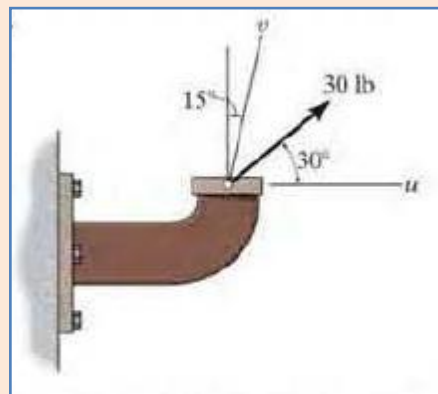


Ans:  $F_R = 666 \text{ N}$

**Exercise 2.7:**

Resolve the 30 lb force into components along the  $u$  and  $v$  axes and determine the magnitude of each of these components.

Fig 2-17

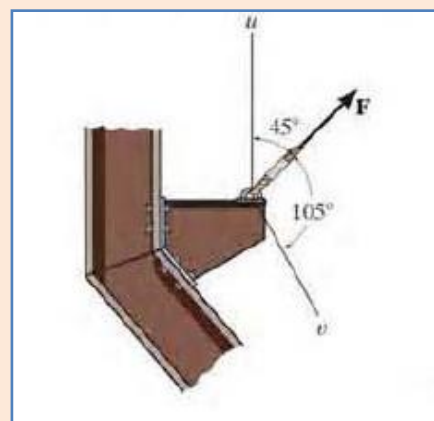


Ans:  $F_u = 22.0 \text{ lb}$      $F_v = 15.5 \text{ lb}$

**Exercise 2.8:**

If force  $\vec{F}$  is to have a component along the  $u$  axis of  $F_u = 6 \text{ kN}$ , determine the magnitude of  $\vec{F}$  and the magnitude of its component  $F_v$  along the  $v$  axis.

Fig 2-18

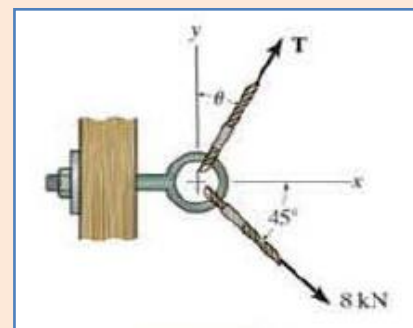


Ans:  $F = 3.11 \text{ kN}$      $F_v = 4.39 \text{ kN}$

**Exercise 2.9:**

If  $\theta = 60^\circ$  and  $T = 5 \text{ kN}$ , determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive  $x$  axis.

Fig 2-19

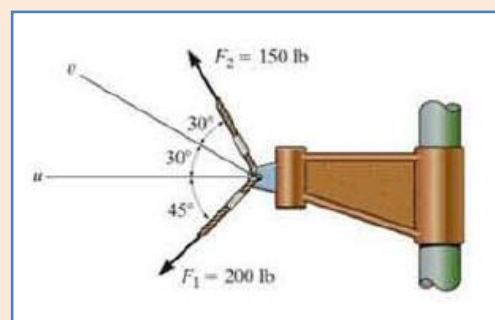


Ans:  $F_R = 10.47 \text{ kN}$      $\phi = 17.5^\circ$

**Exercise 2.10:**

Resolve  $\vec{F}_1$  into components along  $u$  and  $v$  axes and determine the magnitudes of these components.

Fig 2-20



Ans:  $F_u = 386 \text{ lb}$

$F_v = 283 \text{ lb}$

**Exercise 2.11:**

Resolve  $\vec{F}_2$  into components along  $u$  and  $v$  axes, and determine the magnitudes of these components.

See Fig 2-20

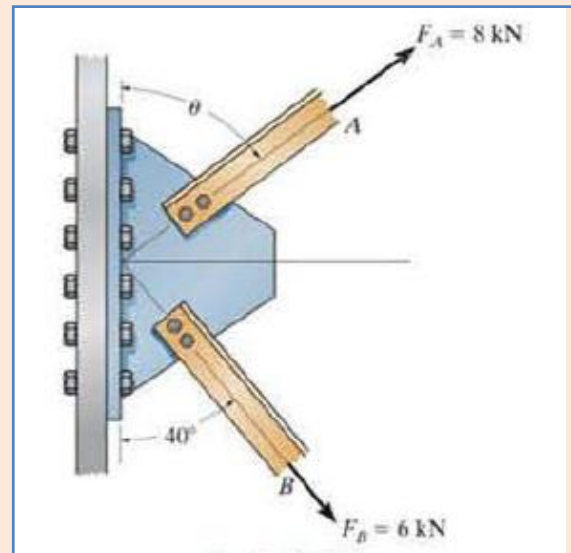
Ans:  $F_u = 150 \text{ lb}$

$F_v = 260 \text{ lb}$

**Exercise 2.12:**

The plate is subjected to the two forces at  $A$  and  $B$  as shown. If  $\theta = 60^\circ$  determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.

Fig 2-21



Ans:  $F_R = 10.8 \text{ kN}$

$\phi = 3.16^\circ$

**Exercise 2.13:**

Determine the angle of  $\theta$  for connecting member  $A$  to the plate so that the resultant force of  $\vec{F}_A$  and  $\vec{F}_B$  is directed horizontally to the right. Also what is the magnitude of the resultant force?

See Fig 2-21

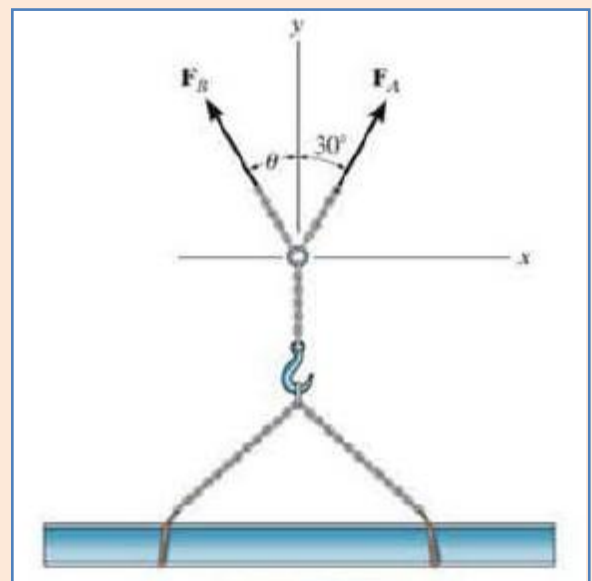
Ans:  $\theta = 54.9^\circ$

$F_R = 10.4 \text{ kN}$

**Exercise 2.14:**

The beam is to be hoisted using two chains. If the resultant force is to be 600 N directed along the positive  $y$  axis, determine the magnitudes of forces  $\vec{F}_A$  and  $\vec{F}_B$  acting on each chain and the angle  $\theta$  of  $\vec{F}_B$  so that the magnitude of  $\vec{F}_B$  is minimum,  $\vec{F}_A$  act at  $30^\circ$  from the  $y$  axis as shown.

Fig 2-22



Ans:  $F_A = 520 \text{ N}$

$F_B = 300 \text{ N}$

**2.4 addition of a system of coplanar forces**

When a force is resolved into two components along the x and y axes the components are then called **rectangular components**.

The rectangular components of force  $F$  shown in Fig 2.23 are found using the parallelogram law, so that

$$\vec{F} = \vec{F}_x + \vec{F}_y$$

$$F_x = F \cos \theta$$

$$F_y = F \sin \theta$$

التمثيل هنا يكون بتحويلها إلى  
مركبات سينية وصادية على  
مستوى المحاور الديكارتية

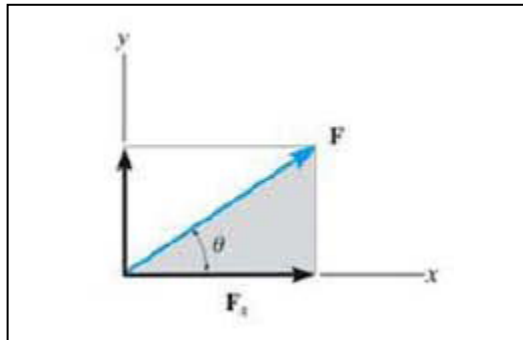


Fig 2-23

instead of using the angle  $\theta$ , the direction of  $\vec{F}$  can also be defined using a small "slope" triangle, such as shown in fig 2.24

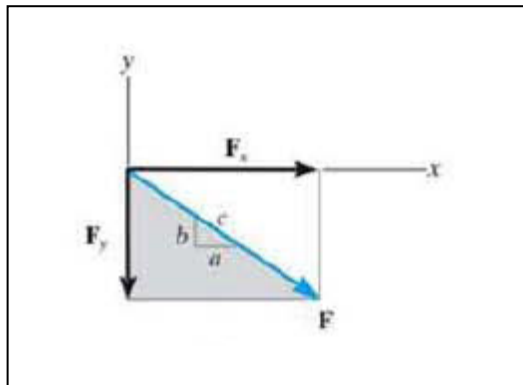


Fig 2-24

يمكن استغلال المثلث الصغير  
المقترن بالقوة لمعرفة مركبات  
القوة.

$$\frac{F_x}{F} = \frac{a}{c} \Rightarrow F_x = F \left( \frac{a}{c} \right) \quad \text{And} \quad \frac{F_y}{F} = \frac{b}{c} \Rightarrow F_y = \left( \frac{b}{c} \right) F$$

It is also possible to represent the x and y components of a force in terms of **Cartesian unit vectors**  $\mathbf{i}$  and  $\mathbf{j}$  (Fig 2.25).

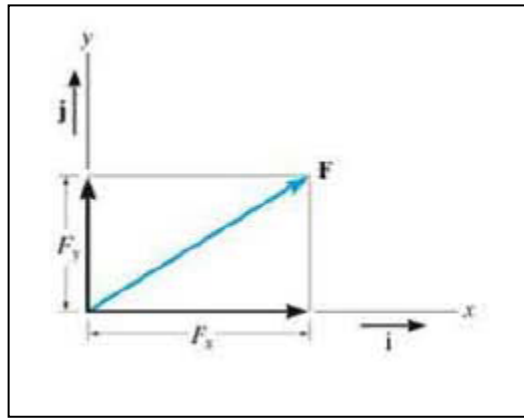


Fig 2-25

We can express  $\vec{F}$  as a **Cartesian vector**.

$$\vec{F} = F_x \vec{i} + F_y \vec{j}$$

In **coplanar force** resultant case, **each force is resolved** into its x and y components, and then the respective components are added using **scalar algebra** since they are collinear. For example, consider the three concurrent forces in Fig 2.26.

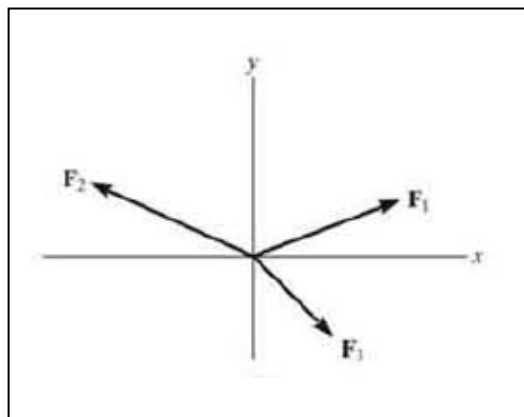


Fig 2-26

Each force is represented as a Cartesian vector.

$$\vec{F}_1 = F_{1x} \vec{i} + F_{1y} \vec{j}$$

$$\vec{F}_2 = -F_{2x} \vec{i} + F_{2y} \vec{j}$$

$$\vec{F}_3 = F_{3x} \vec{i} - F_{3y} \vec{j}$$

The vector resultant is therefore.

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (F_{1x} + F_{2x} + F_{3x}) \vec{i} + (F_{1y} + F_{2y} + F_{3y}) \vec{j}$$

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (F_{Rx}) \vec{i} + (F_{Ry}) \vec{j}$$

We can **represent** the components of the resultant force of any number of coplanar forces symbolically by the **algebraic sum** the **x** and **y** components of all the forces.

$$F_{Rx} = \Sigma F_x$$

$$F_{Ry} = \Sigma F_y$$

Once these components are determined, they may be sketched along the x and y axes with their proper sense of direction, and the resultant force can be determined from vector addition as shown in Fig 2-27.

The **magnitude of  $\vec{F}_R$**  is then found from the by **Pythagorean theorem**:  
that is

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1} \left| \frac{F_{Ry}}{F_{Rx}} \right|$$

كمية وزاوية المحصلة  
يكون باستعمال نظرية  
بيثاغورث للمثلثات  
القائمة

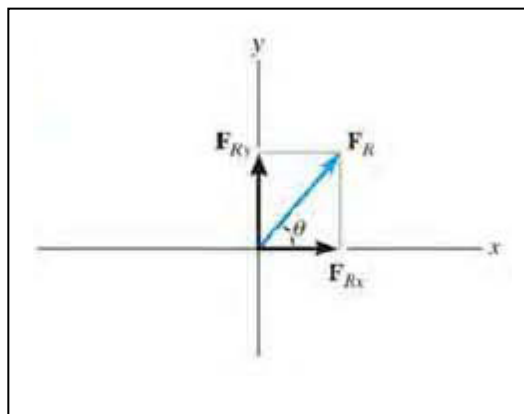
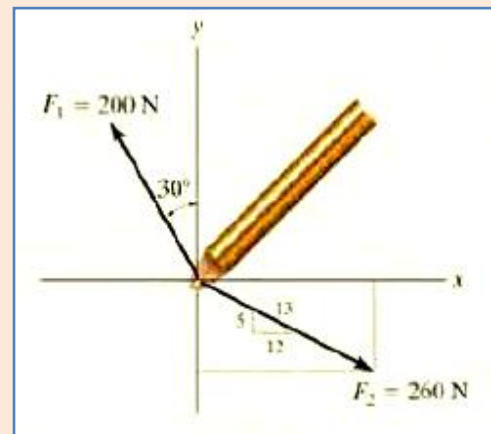


Fig 2-27

**Exercise 2.15:**

Determine the x and y components of  $\vec{F}_1$  and  $\vec{F}_2$  acting on the boom shown in Fig 2.28 express each force as a Cartesian vector.

Fig 2-28



$$\text{Ans: } F_{1x} = -100 \text{ N} \quad F_{1y} = 173 \text{ N} \quad F_{2x} = 240 \text{ N} \quad F_{2y} = -100 \text{ N}$$

$$\vec{F}_1 = (-100\mathbf{i} + 173\mathbf{j}) \text{ N} \quad \vec{F}_2 = (240\mathbf{i} - 100\mathbf{j}) \text{ N}$$

**Exercise 2.16:**

The link in Fig 2.29 is subjected to two forces  $\vec{F}_1$  and  $\vec{F}_2$ . Determine the magnitude and direction of the resultant force.

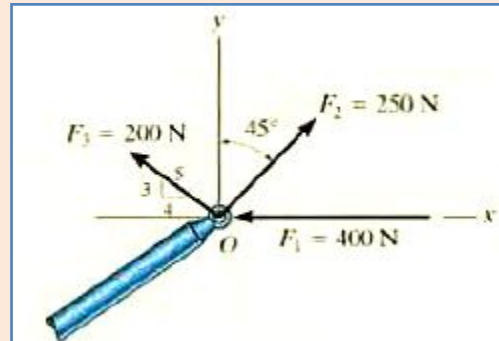
Fig 2-29

$$\text{Ans: } F_R = 629 \text{ N} \quad \theta = 67.9^\circ$$

**Exercise 2.17:**

The end of boom O in Fig 2.30 is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.

Fig 2-30

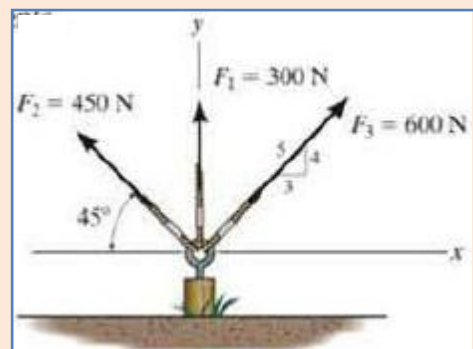


$$\text{Ans: } F_R = 485 \text{ N} \quad \theta = 37.8^\circ$$

**Exercise 2.18:**

Resolve each force acting on the post into its x and y components.

Fig 2-31

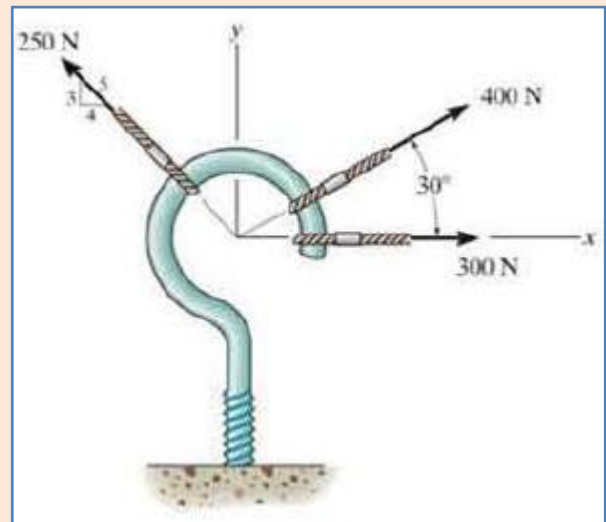


$$\text{Ans: } F_{1x} = 0 \text{ N} \quad F_{1y} = 300 \text{ N} \quad F_{2x} = -318 \text{ N} \quad F_{2y} = 318 \text{ N} \quad F_{3x} = 360 \text{ N} \quad F_{3y} = 480 \text{ N}$$

**Exercise 2.19:**

Determine the magnitude and direction of the resultant force.

Fig 2-32



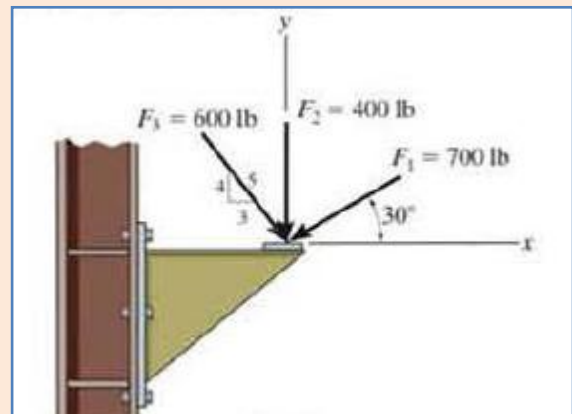
$$\text{Ans: } F_R = 567 \text{ N}$$

$$\theta = 38.1^\circ$$

**Exercise 2.20:**

Determine the magnitude of the resultant force acting on the corbel and its direction  $\theta$  measured counterclockwise from the x axis.

Fig 2-33



$$\text{Ans: } F_R = 1254 \text{ lb}$$

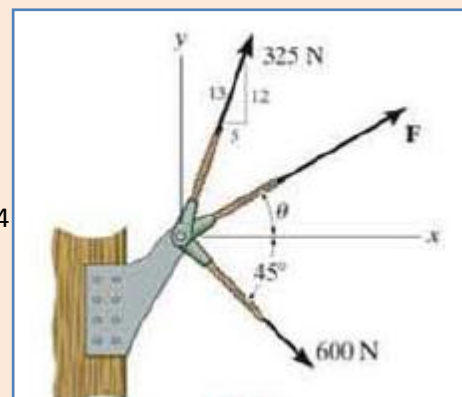
$$\phi = 78.68^\circ$$

$$\theta = 180 + \phi = 259^\circ$$

**Exercise 2.21:**

If the resultant force acting on the bracket is to be 750 N directed along the positive x axis, determine the magnitude of  $\vec{F}$  and its direction  $\theta$ .

Fig 2-34



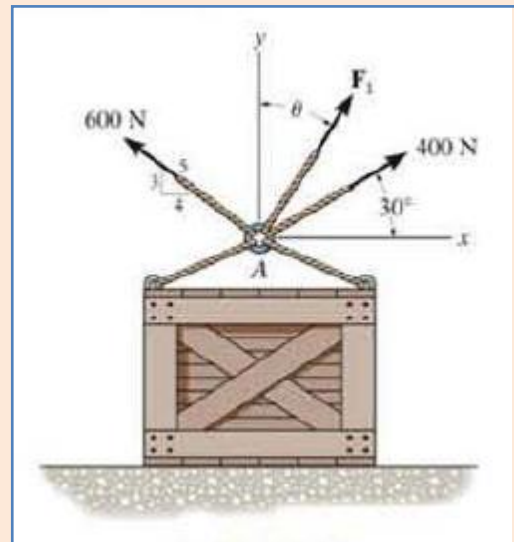
$$\text{Ans: } \theta = 31.76^\circ \quad F = 236 \text{ N}$$



**Exercise 2.22:**

Determine the magnitude and direction measured counterclockwise from the positive  $x$  axis of the resultant force of the three forces acting on the ring A. Take  $F_1 = 500\text{ N}$  and  $\theta = 20^\circ$ .

Fig 2-35

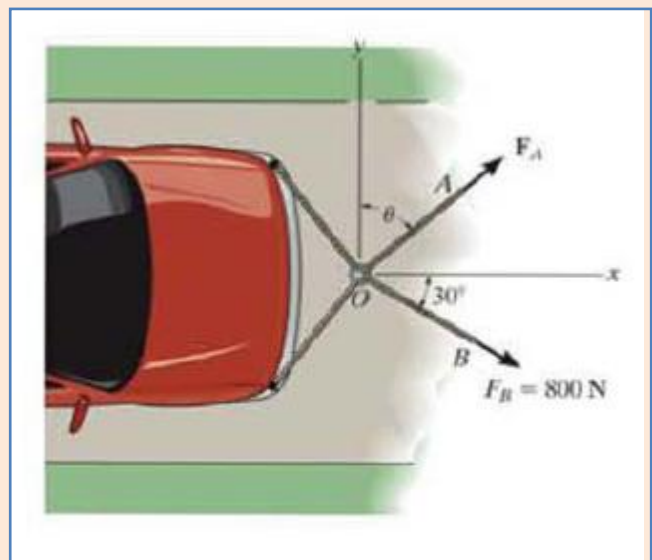


Ans:  $F_R = 1.03\text{ kN}$   $\theta = 87.9^\circ$

**Exercise 2.23:**

Determine the magnitude and direction measured counterclockwise from the positive  $x$  axis of the resultant force acting on the ring at O, if  $F_A = 750\text{ N}$  and  $\theta = 45^\circ$ .

Fig 2-36

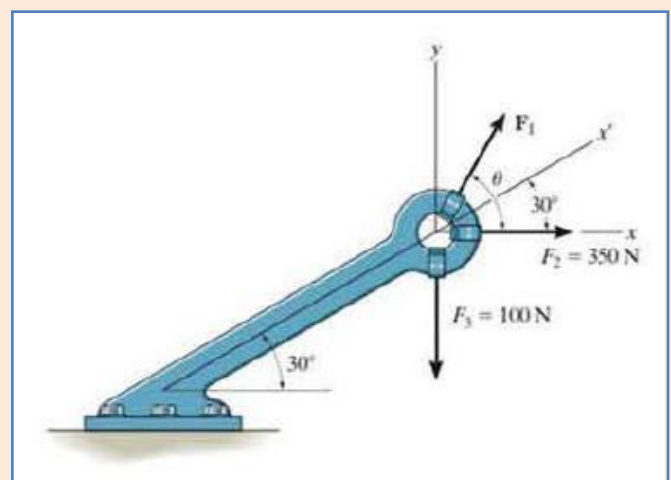


Ans:  $F_R = 1.23\text{ kN}$   $\theta = 6.08^\circ$

**Exercise 2.24:**

Express each of the three forces acting on the bracket in Cartesian vector form with respect to the  $x$  and  $y$  axes. Determine the magnitude and direction  $\theta$  of  $\vec{F}_1$  so that the resultant force is directed along the positive  $x'$  axis and has a magnitude of  $F_R = 600\text{ N}$ .

Fig 2-37



Ans:  $F_1 = 434.5\text{ N}$   $\theta = 67^\circ$

**2.5 Cartesian vectors**

A vector  $\vec{A}$  may have three rectangular components along the  $x$ ,  $y$ ,  $z$  coordinate axes and is represented by the vector sum of its **three rectangular components** (Fig 2-38).

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

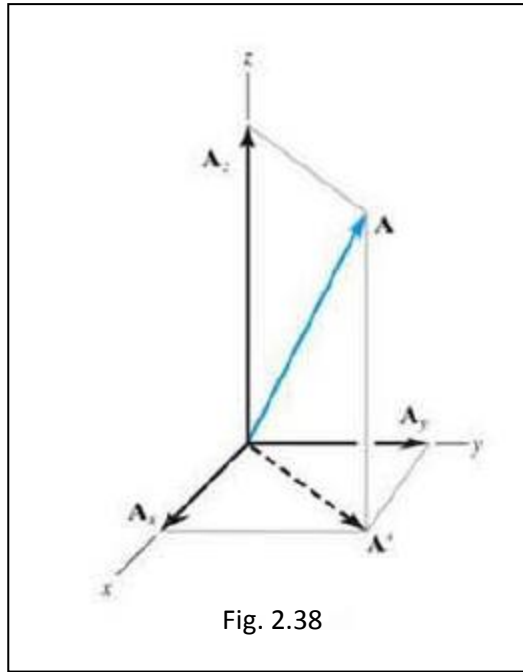


Fig. 2.38

القوى المؤثرة في  
البعد الثالث  
لها مركبات حسب  
المحاور  $x$  و  $y$  و  $z$

In three dimensions, the set of Cartesian unit  $\vec{i}$ ,  $\vec{j}$ ,  $\vec{k}$  is used to designate the directions of the  $x$ ,  $y$ ,  $z$  axes, respectively. The positive Cartesian unit vectors are shown in Fig 2-39.

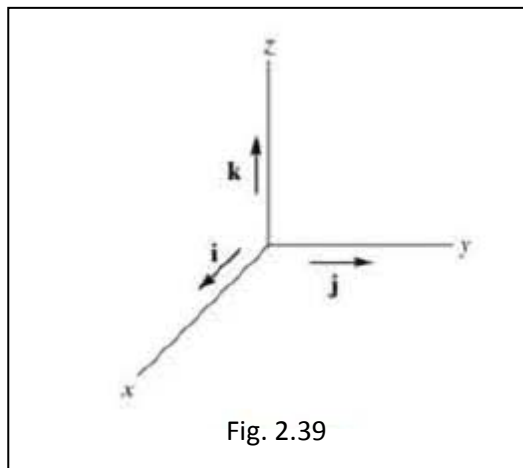


Fig. 2.39

We can write  $\vec{A}$  in **Cartesian vector** form as

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

The **magnitude** of  $\vec{A}$  is expressed in Cartesian vector form as

نستخدم متجهات الوحدة الديكارتية  $\vec{i}$   
و  $\vec{j}$  و  $\vec{k}$  لتعريف كمية المتجه  $\vec{A}$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

The **direction** of  $\vec{A}$  is defined by the **coordinate direction** angles  $\alpha$ ,  $\beta$ , and  $\gamma$  (Fig 2.40).

$$\cos \alpha = \frac{A_x}{A} \quad \cos \beta = \frac{A_y}{A} \quad \cos \gamma = \frac{A_z}{A}$$

With

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

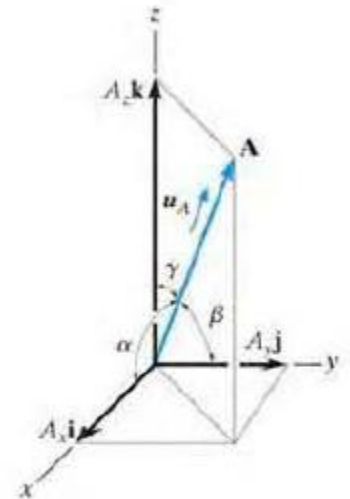


Fig. 2.40

The addition (or subtraction) of two or more vectors are greatly simplified in terms of their Cartesian components. For example, the resultant  $\vec{R}$  in Fig 2.41 is written as

$$\vec{R} = (A_x + B_x)\vec{i} + (A_y + B_y)\vec{j} + (A_z + B_z)\vec{k}$$

حالة جمع متجهين في  
الفضاء

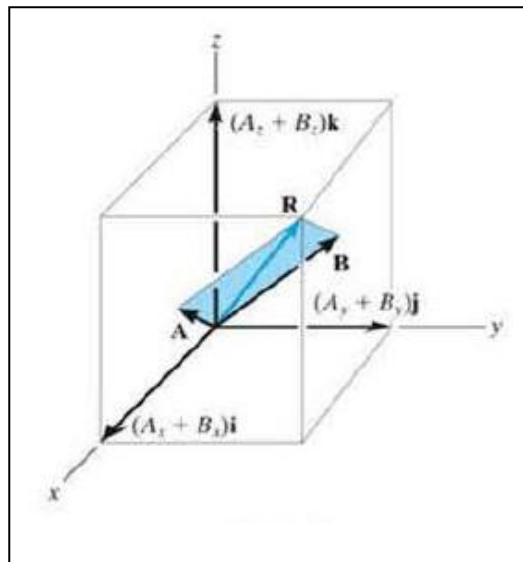


Fig. 2.41

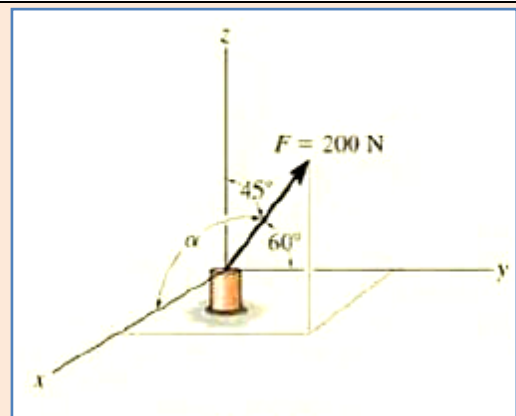
If this is generalized and applied to a system of several concurrent forces, then the force resultant is the vector sum of all the forces in the system and can be written as

$$\vec{F}_R = \Sigma \vec{F} = \Sigma F_x \vec{i} + \Sigma F_y \vec{j} + \Sigma F_z \vec{k}$$

**Exercise 2.25:**

Express the force  $\vec{F}$  shown in Fig 2.38 as a cartesian vector.

Fig 2-38

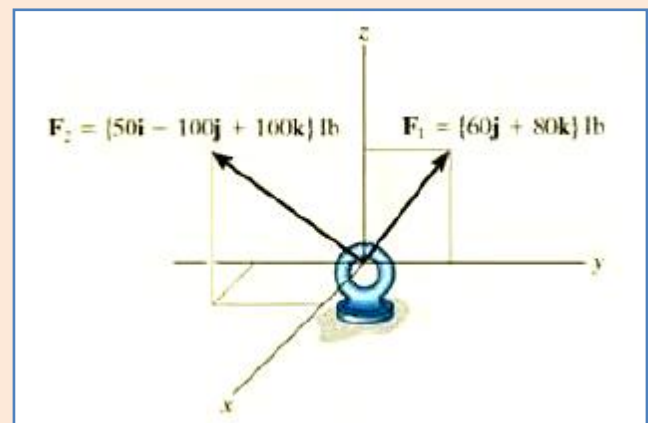


$$\text{Ans: } \vec{F} = \{100 \hat{i} + 100 \hat{j} + 141.4 \hat{k}\} \text{ N}$$

**Exercise 2.26:**

Determine the magnitude and the coordinate direction angles of the resultant force acting on the ring in Fig 2-39

Fig 2-39

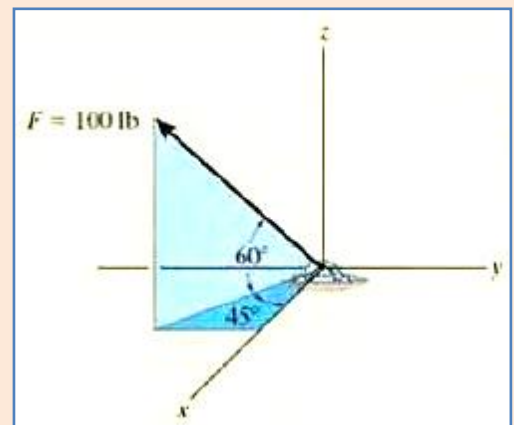


$$\text{Ans: } F_R = 191 \text{ lb} \quad \cos \alpha = 0.2617 \quad \alpha = 74.8^\circ \quad \cos \beta = -0.2094 \quad \beta = 102^\circ \\ \cos \gamma = 0.9422 \quad \gamma = 19.6^\circ$$

**Exercise 2.27:**

Express the force  $\vec{F}$  shown in Fig 2.40 as a Cartesian vector, And determine its coordinate direction angles.

Fig 2-40

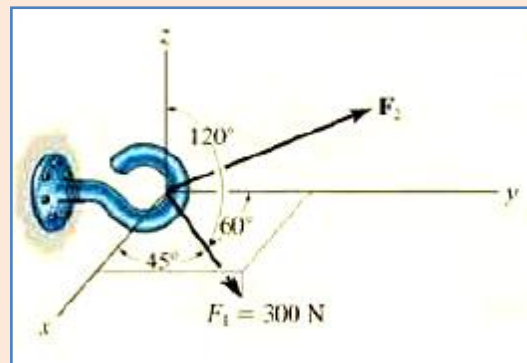


$$\text{Ans: } \vec{F} = \{35.4 \hat{i} - 35.4 \hat{j} + 86.6 \hat{k}\} \text{ lb} \quad \alpha = 69.3^\circ \quad \beta = 111^\circ \quad \gamma = 30^\circ$$

**Exercise 2.28:**

Two forces act on the hook in Fig 2-41, specify the magnitude of  $\vec{F}_2$  and its coordinate direction angles of  $\vec{F}_2$  that the resultant force  $\vec{F}_R$  acts along the positive y axis and has magnitude of 800 N.

Fig 2-41

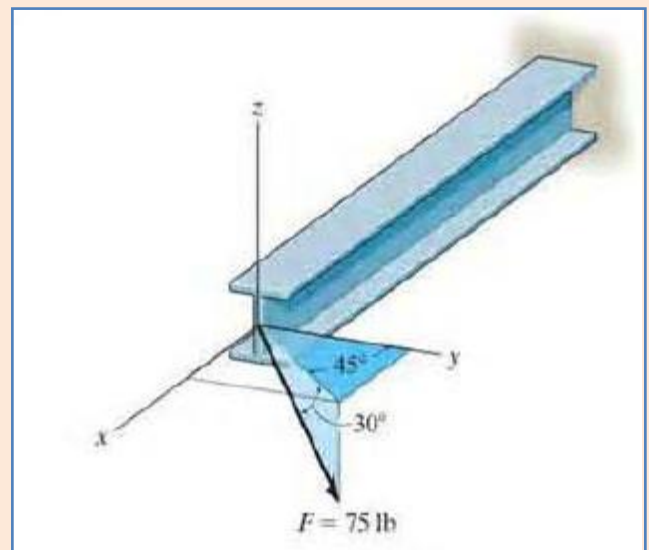


$$\begin{aligned} \text{Ans: } F_2 &= 700 \text{ N} & \cos \alpha_2 &= \frac{-212.1}{700} \Rightarrow \alpha_2 = 108^\circ \\ & & \cos \beta_2 &= \frac{650}{700} \Rightarrow \beta_2 = 21.8^\circ \\ & & \cos \gamma_2 &= \frac{150}{700} \Rightarrow \gamma_2 = 77.6^\circ \end{aligned}$$

**Exercise 2.29:**

Determine its coordinate direction angles of the force.

Fig 2-42

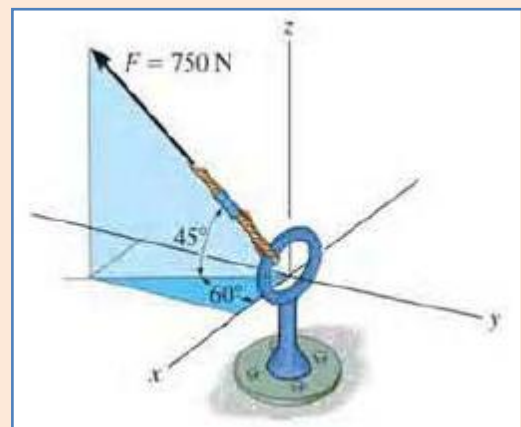


$$\text{Ans: } \alpha = 52.2^\circ \quad \beta = 52.2^\circ \quad \gamma = 120^\circ$$

**Exercise 2.30:**

Express the force as a Cartesian vector.

Fig 2-43

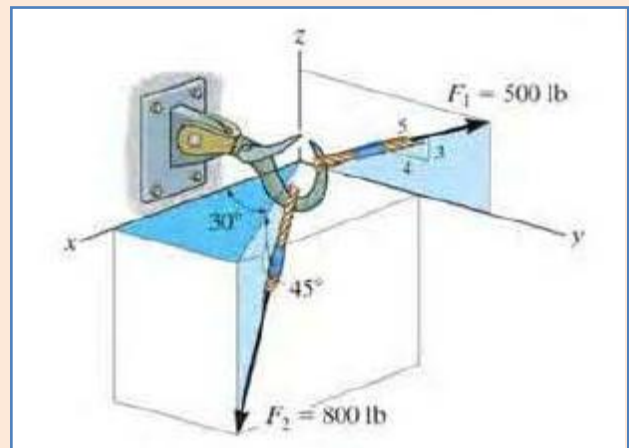


$$\text{Ans: } \vec{F} = \{265\vec{i} - 459\vec{j} + 530\vec{k}\} \text{ N}$$

**Exercise 2.31:**

Determine the resultant force acting on the hook.

Fig 2-45



$$\text{Ans: } \vec{F}_R = \vec{F}_1 + \vec{F}_2 = \{490\vec{i} + 683\vec{j} - 266\vec{k}\} \text{ lb}$$

**Exercise 2.32:**

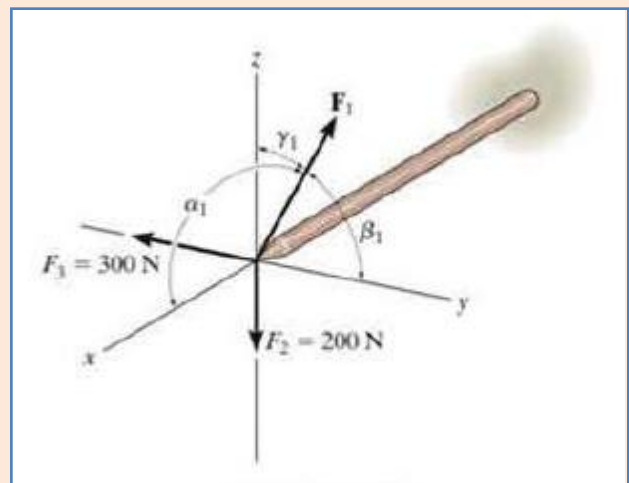
The mast is subject to the three forces shown.

Determine the coordinate direction

angles  $\alpha_1, \beta_1, \gamma_1$  of  $\vec{F}_1$  so that the resultant force acting on the mast is  $\vec{F}_R = \{350\vec{i}\} \text{ N}$ .

Take  $F_1 = 500 \text{ N}$ .

Fig 2-46



$$\text{Ans: } \alpha_1 = 45.6^\circ \quad \beta_1 = 53.1^\circ \quad \gamma_1 = 66.4^\circ$$

**Exercise 2.33:**

The mast is subject to the three forces shown. Determine the coordinate direction angles  $\alpha_1, \beta_1, \gamma_1$  of  $\vec{F}_1$  so that the resultant force acting on the mast is zero (see Fig. 2.46).

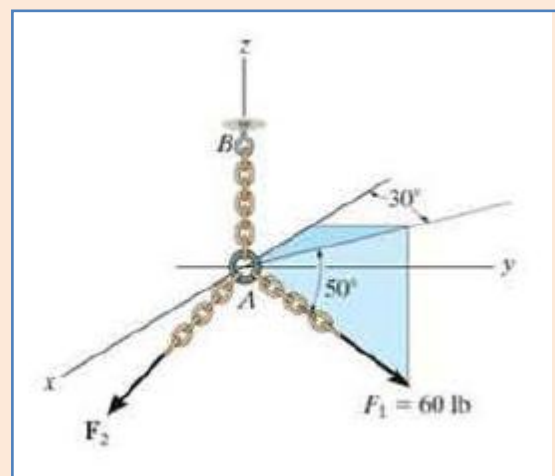
$$\text{Ans: } \alpha_1 = 90^\circ \quad \beta_1 = 53.1^\circ \quad \gamma_1 = 66.4^\circ$$

**Exercise 2.34:**

The two forces  $\vec{F}_1$  and  $\vec{F}_2$  acting at A have a resultant force of  $\vec{F}_R = \{-100\vec{k}\} \text{ lb}$ .

Determine the magnitude and coordinate direction angles of  $\vec{F}_2$ .

Fig 2-47



$$\text{Ans: } F_2 = 66.4 \text{ lb} \quad \alpha = 59.8^\circ \quad \beta = 107^\circ \quad \gamma = 144^\circ$$

## 2.6 Position Vectors

In the more general case, the position vector may be directed from point A to point B in space, Fig. 2-48. This vector is also designated by the **symbol**  $\mathbf{r}$ . As a matter of convention, we will sometimes refer to this vector with two subscripts to indicate from and to the point where it is directed. Thus,  $\mathbf{r}$  can also be designated as  $\mathbf{r}_{AB}$ . Also, note that  $\mathbf{r}_A$  and  $\mathbf{r}_B$  in Fig. 2-48, are referenced with only one subscript since they extend from the origin of coordinates.

From Fig. 2-48, by the **head-to-tail vector** addition, using the **triangle** We require:

$$\vec{r}_A + \vec{r} = \vec{r}_B$$

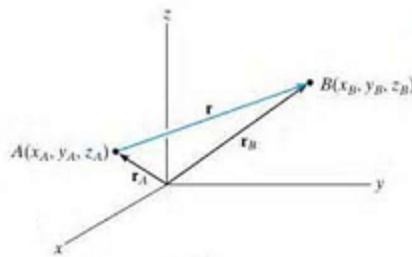
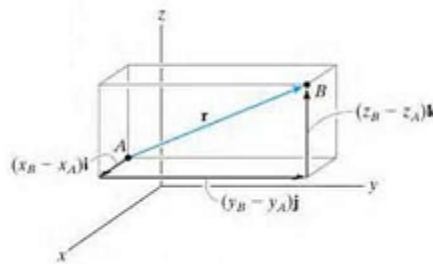


Fig 2-48



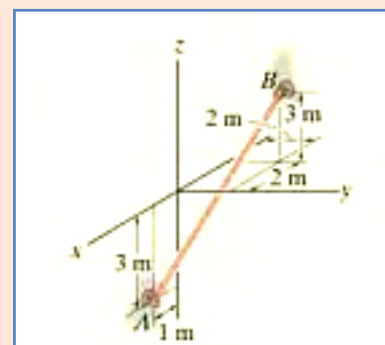
Solving for  $\vec{r}$  and expressing  $\vec{r}_A$  and  $\vec{r}_B$  in Cartesian vector form yields

$$\vec{r} = \vec{r}_B - \vec{r}_A = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$

### Exercise 2.35:

An elastic rubber band is attached to points A and B as shown in Fig 2-49. Determine its length and its direction measured from A towards B.

Fig 2-49



$$\text{Ans: } r = 7 \text{ m}$$

$$\alpha = 115^\circ$$

$$\beta = 73.4^\circ$$

$$\gamma = 31^\circ$$

**2.7 Dot Product**

the dot Product of vectors  $\vec{A}$  and  $\vec{B}$  written  $\vec{A} \cdot \vec{B}$  and read  $\vec{A}$  dot  $\vec{B}$  is defined as the product of the magnitudes of A and B and the cosine of the angle  $\theta$  between their tails (Fig 2.50).

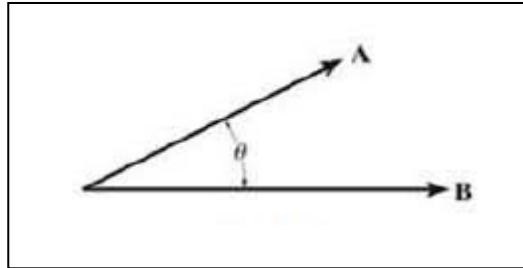
expressed in equation form.

$$\vec{A} \cdot \vec{B} = AB \cos \theta \quad (2.1)$$

Where

$$0^\circ \leq \theta \leq 180^\circ$$

Fig 2-50



الضرب القياسي  
لمتجهين ينتج منه  
كمية عددية  
(scalar)

equation 2.1 must be used to find the dot product for any two Cartesian unit vectors.

For example:

$$\vec{i} \cdot \vec{i} = (1)(1) \cos 0^\circ = 1 \quad \vec{j} \cdot \vec{j} = 1 \quad \vec{k} \cdot \vec{k} = 1$$

$$\vec{i} \cdot \vec{j} = (1)(1) \cos 90^\circ = 0 \quad \vec{j} \cdot \vec{k} = 0 \quad \vec{i} \cdot \vec{k} = 0$$

تطبيق قاعدة الضرب  
القياسي على  
متجهات الوحدة

if we want to find the dot product of two general vectors  $\vec{A}$  and  $\vec{B}$  that are expressed in Cartesian vector form, then we have

$$\vec{A} \cdot \vec{B} = (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) (B_x \vec{i} + B_y \vec{j} + B_z \vec{k})$$

$$\vec{A} \cdot \vec{B} = A_x B_x (\vec{i} \cdot \vec{i}) + A_x B_y (\vec{i} \cdot \vec{j}) + A_x B_z (\vec{i} \cdot \vec{k}) + A_y B_x (\vec{j} \cdot \vec{i}) + A_y B_y (\vec{j} \cdot \vec{j}) + A_y B_z (\vec{j} \cdot \vec{k}) + A_z B_x (\vec{k} \cdot \vec{i}) + A_z B_y (\vec{k} \cdot \vec{j}) + A_z B_z (\vec{k} \cdot \vec{k})$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \quad (2.2)$$

We deduce that the angle forces between two vectors can be written as

$$\theta = \cos^{-1} \left( \frac{\vec{A} \cdot \vec{B}}{AB} \right)$$

Where

$$0^\circ \leq \theta \leq 180^\circ$$



we notice that if

$$\vec{A} \cdot \vec{B} = 0 \Rightarrow \theta = \cos^{-1} 0 = 90^\circ$$

so that  $\vec{A}$  will be perpendicular to  $\vec{B}$ .

In the case of line  $a$  as shown in figure 2-51, and if the direction of the line is specified by the unit  $\vec{u}_a$  then since  $u_a = 1$ , we can determine the magnitude of  $\vec{A}_a$  directly from the dot product

$$A_a = A \cos \theta$$

$$\vec{A} \cdot \vec{u}_a = A \cdot 1 \cdot \cos \theta = A \cos \theta \Rightarrow A_a = \vec{A} \cdot \vec{u}_a$$

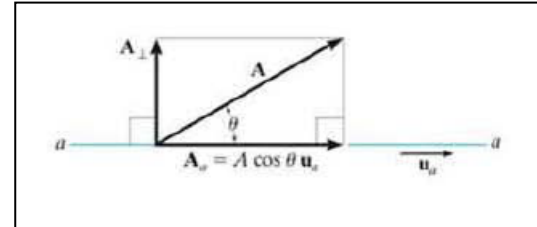


Fig 2-51

Notice that if this result is positive, then  $\vec{A}_a$  has a directional sense which is the same as  $\vec{u}_a$ , whereas if  $A_a$  is a negative scalar, then  $\vec{A}_a$  has the opposite sense of direction  $\vec{u}_a$ . The component  $\vec{A}_a$  represented as a vector is therefore

$$\vec{A}_a = A_a \vec{u}_a$$

The component of  $\vec{A}$  that is perpendicular to line  $a$  ( $\vec{A}_\perp$ ) can also be obtained from Figure 2-51. Therefore

$$A_\perp = A \sin \theta \quad \text{with} \quad \theta = \cos^{-1} \left( \frac{\vec{A} \cdot \vec{u}_a}{A} \right)$$

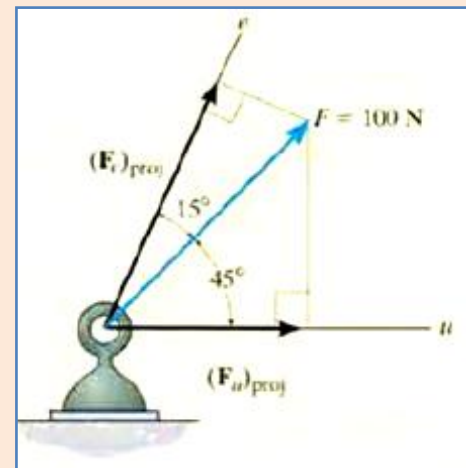
Alternatively as if  $A_a$  is known then by Pythagorean's theorem we can also write

$$A_\perp = \sqrt{A^2 - A_a^2}$$

### **Exercise 2.36:**

Determine the magnitude of the projection of the force  $\vec{F}$  in Fig 2-52 onto the  $u$  and  $v$  axes (we note that these projections are not equal to the magnitudes of the components of force  $\vec{F}$  along  $u$  and  $v$  found from the parallelogram law. They will only equal if the  $u$  and  $v$  axes are perpendicular to another).

Fig 2-52

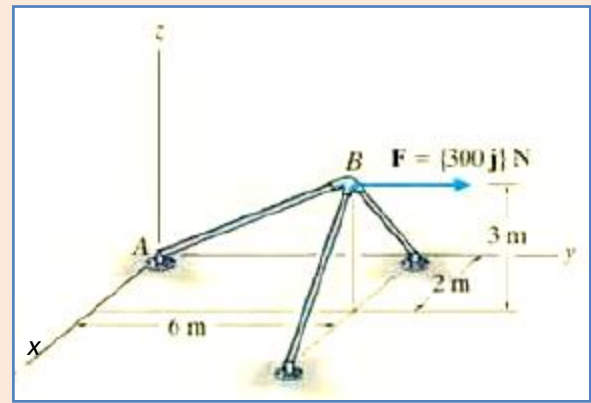


$$\text{Ans: } (F_u)_{proj} = 70.7 \text{ N} \quad (F_v)_{proj} = 96.6 \text{ N}$$

**Exercise 2.37:**

The frame shown in Fig 2-53 is subjected to a horizontal force  $\vec{F} = \{300 \hat{j}\}$ . Determine the magnitude of the components of this force parallel and perpendicular to member AB.

Fig 2-53

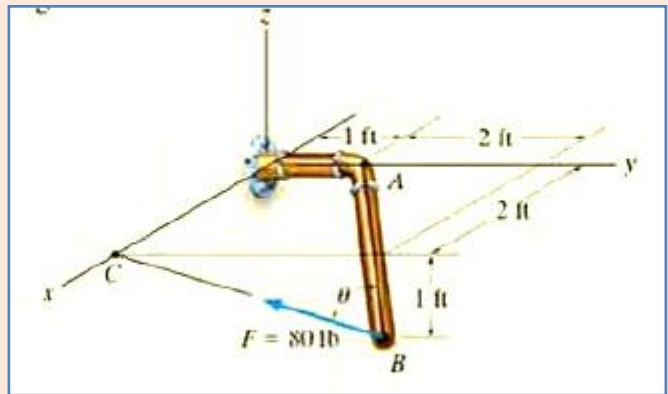


Ans:  $F_{Ab} = 257.1 \text{ N}$       $F_{\perp} = 155 \text{ N}$

**Exercise 2.38:**

The pipe in Fig 2.54 is subjected to the force of  $F = 80 \text{ lb}$ . Determine the angle  $\theta$  between  $\vec{F}$  and the pipe segment BA and the projection of  $\vec{F}$  Along this segment.

Fig 2-54

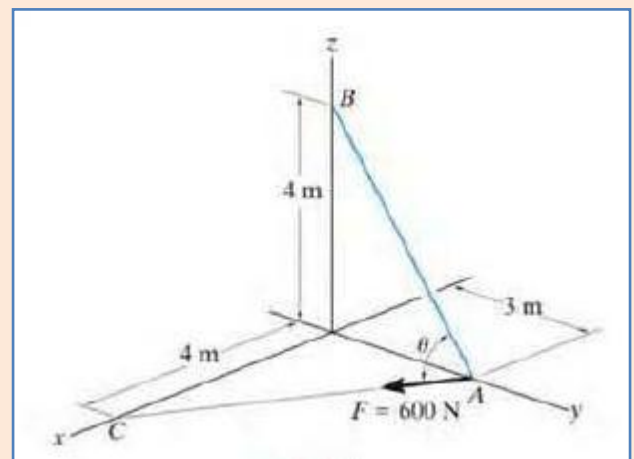


Ans:  $\theta = 42.5^\circ$       $F_{BA} = 59 \text{ lb}$

**Exercise 2.39:**

Determine the angle  $\theta$  between the force and line AB.

Fig 2-55



Ans:  $\theta = 68.9^\circ$

# Chapter

Three

## Equilibrium Of a Particle

أخي الطالب عليك آداب جمّة يجب أن تلتزم بها ولا تتخلّى عنها وهي أن تكون إرادتك بطلب العلم هي  
الإخلاص لله ثم لوطنك، وأن تتخلق بمحاسن الأخلاق و تتحلّى بمكارم الآداب و تتصف بالصفات  
الحميدة و تتجمل بالشيم المرضية وأن تتجنب الحسد والرياء والعجب والكبر

## Equilibrium of a Particle

### 3.1 Condition for the equilibrium of a particle.

A particle is said to be in equilibrium if it **remains at rest** if **originally at rest**, or **has a constant velocity** if **originally in motion**. to maintain equilibrium, it is necessary to satisfy Newton's first law of motion which requires the resultant force acting on a particle to be equal to zero. This condition may be stated mathematically as

$$\sum \vec{F} = 0 \quad (3.1)$$

Where  $\sum \vec{F}$  is the vector sum of all the forces acting on the particle.

شرط توازن جسم هو  
أن تكون محصلة القوى  
الخارجية المؤثرة على  
الجسم تساوي صفر.

### 3.2 The free body diagram

A **drawing** that **shows** the particle with **all the forces** that act on it is called a **free body diagram (FBD)**.

We will consider **a springs connections** often encountered in particle equilibrium problems.

مخطط الجسم الحر هو تمثيل  
تصويري يستخدم لتحليل  
القوى المؤثرة على الجسم  
الحر.

**Springs:** If a linearly elastic spring of undeformed length  $l_0$  is used to support a particle, **the length of the spring will change in direct proportion to the force  $F$  acting on it**, Fig 3.1. A **characteristic** that defines the **elasticity of a spring** is the **spring constant** or **stiffness  $k$** . The magnitude of force exerted on a linearly elastic spring is stated as

$$F = k s$$

Where

$$s = l - l_0$$

النوابض ذات الملف  
اللزوني هي عناصر  
مرنة تقاوم القوى  
الضاغطة المطبقة باتجاه  
محورها، وهي تخضع  
لقانون هوك للمرونة  
الذي يعتبر بأن الامتداد  
الناتج يتناسب مباشرة  
مع الحمل.

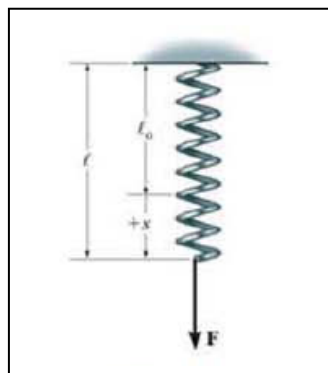
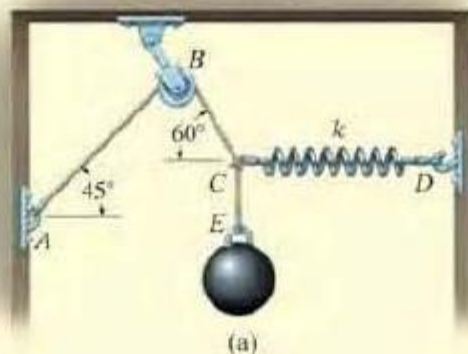


Fig 3.1

$s$  هو الفرق بين موضع الجسم الجديد و  
موقعه الأصلي.  $k$  هو ثابت المرونة.

The following example shows a drawing of the free body diagram of a sphere.

The sphere has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the sphere, the cord  $CE$ , and the knot at  $C$ .



يظهر مخطط الجسم الحر كل  
قوى التلامس المؤثرة على  
الجسم الكروي و الحبل و  
العقدة C.

$F_{CE}$  (Force of cord  $CE$  acting on sphere)



58.9 N (Weight or gravity acting on sphere)

(b)

**Sphere.** By inspection, there are only two forces acting on the sphere, namely, its weight,  $6 \text{ kg} (9.81 \text{ m/s}^2) = 58.9 \text{ N}$ , and the force of cord  $CE$ . The free-body diagram is shown in Fig.

$F_{EC}$  (Force of knot acting on cord  $CE$ )

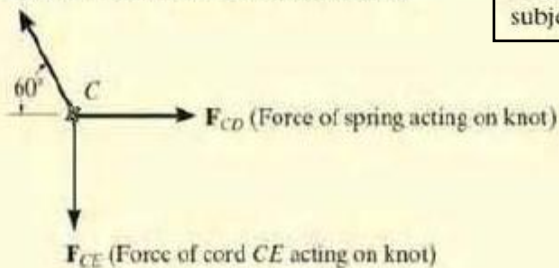


$F_{CE}$  (Force of sphere acting on cord  $CE$ )

(c)

**Cord  $CE$ .** When the cord  $CE$  is isolated from its surroundings, its free-body diagram shows only two forces acting on it, namely, the force of the sphere and the force of the knot, Fig. 3-3c. Notice that  $F_{CE}$  shown here is equal but opposite to that shown in Fig. 3-3b, a consequence of Newton's third law of action-reaction. Also,  $F_{CE}$  and  $F_{EC}$  pull on the cord and keep it in tension so that it doesn't collapse. For equilibrium,  $F_{CE} = F_{EC}$ .

$F_{CBA}$  (Force of cord  $CBA$  acting on knot)



(d)

**Knot.** The knot at  $C$  is subjected to three forces. They are caused by the cords  $CBA$  and  $CE$  and the spring  $CD$ . As required, the free-body diagram shows all these forces labeled with their magnitudes and directions. It is important to recognize that the weight of the sphere does not directly act on the knot. Instead, the cord  $CE$  subjects the knot to this force.

**3.3 Coplanar force systems**

If a particle is subjected to a system of coplanar forces as in Fig 3-2, then each force can be resolved into its  $\mathbf{i}$  and  $\mathbf{j}$  components. **For equilibrium these forces must sum to produce a zero free resultant.**

$$\sum \vec{F} = 0$$

$$\sum F_x \mathbf{i} + \sum F_y \mathbf{j} = 0$$

Hence

$$\sum F_x = 0$$

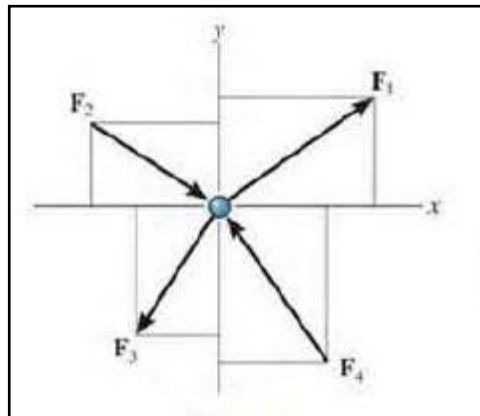
$$\sum F_y = 0$$

بعد تحليل القوى إلى مركباتها الأفقية و  
العمودية يمكن كتابة معادلة الاتزان  
الجسم في اتجاه كل محور.

مجموع القوى في اتجاه المحور  $x$  تساوي  
0.

مجموع القوى في اتجاه المحور  $y$  تساوي  
0.

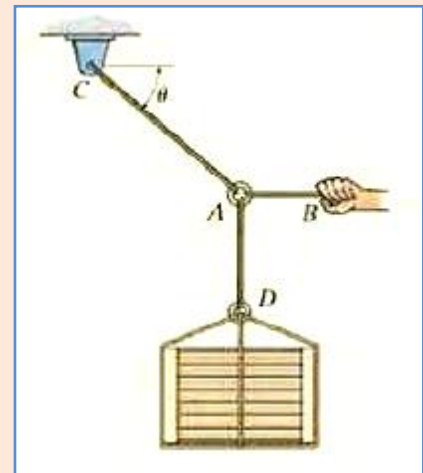
Fig 3-2



**Exercise 3.1:**

Determine the tension in cables BA and BC necessary to support the 60 kg cylinder in fig 3-3.

Fig 3-3

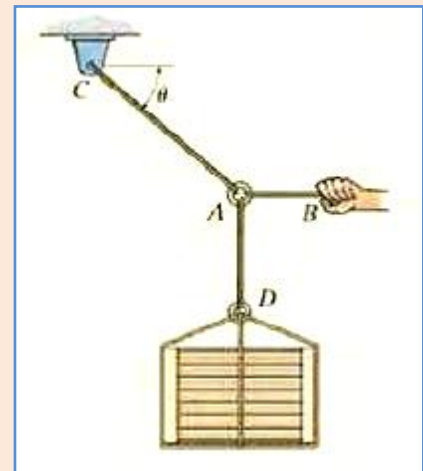


Ans:  $T_C = 476 \text{ N}$        $T_A = 420 \text{ N}$

**Exercise 3.2:**

The 200 kg crate in fig 3.4 a is supported using the ropes AB and AC. Each rope can withstand a maximum force of 10 kN, before it breaks. If AB always remains horizontally, determine the smallest angle  $\theta$  to which the crate can be suspended before one of the ropes breaks.

Fig 3-4

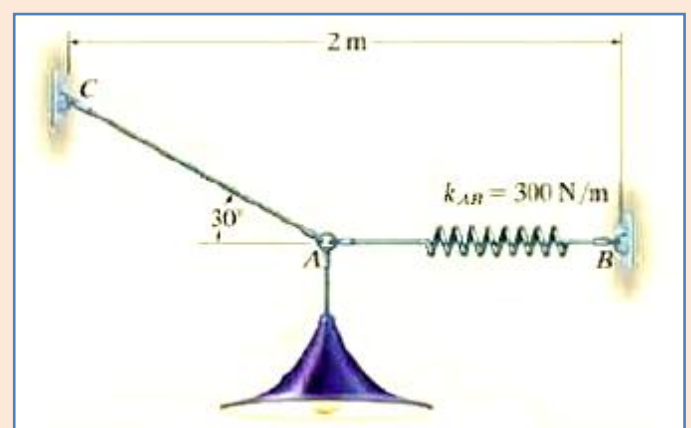


Ans:  $\theta = 11.31^\circ$        $F_B = 9.81 \text{ N}$

**Exercise 3.3:**

Determine the required length of AC in fig 3.5 so that the 8 kg lamp can be suspended in the position shown. The undeformed length of spring AB is  $l'_{AB} = 0.4 \text{ m}$ , and the spring has a stiffness of  $k_{AB} = 300 \text{ N/m}$ .

Fig 3-5



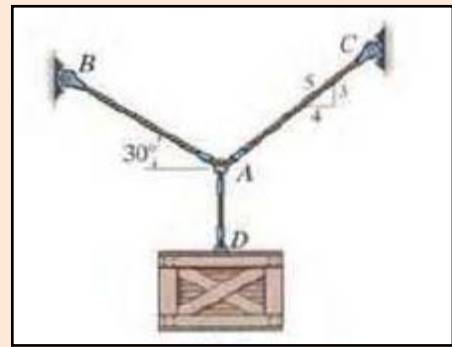
Ans:  $l_{AC} = 1.32 \text{ m}$



**Exercise 3.4:**

The crate has a weight of 550 lb. Determine the force in each supporting cable.

Fig 3-6



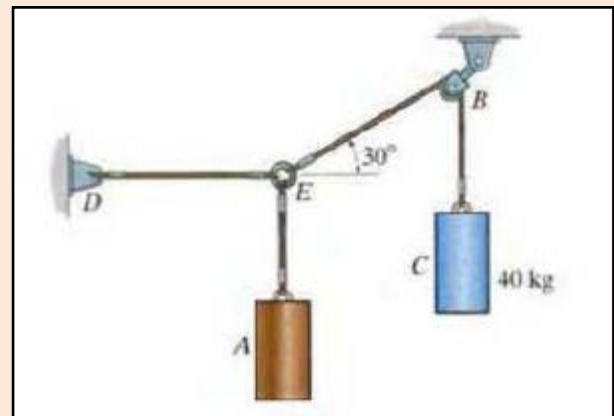
Ans:  $F_{AB} = 478 \text{ lb}$

$F_{AC} = 518 \text{ lb}$

**Exercise 3.5:**

If the mass of cylinder C is 40 kg, determine the mass of cylinder A in order to hold the assembly in the position shown.

Fig 3-7



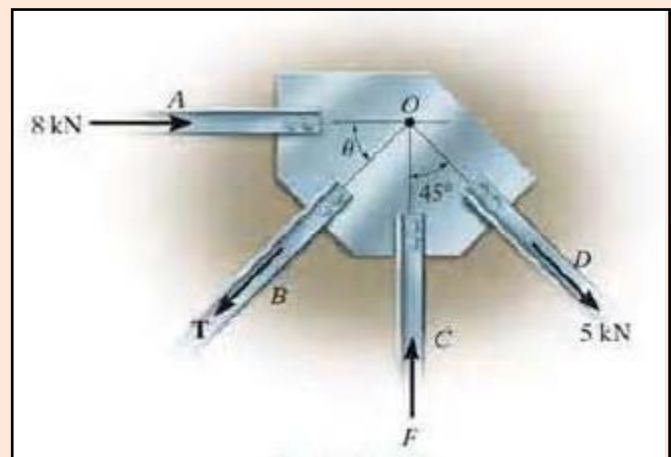
Ans:  $\sum F_y = 0$

$m_a = 20 \text{ kg}$

**Exercise 3.6:**

The members of a truss are connected to the gusset plate. If the forces are concurrent at point O, determine the magnitudes of  $\vec{F}$  and  $\vec{T}$  for equilibrium. Take  $\theta = 30^\circ$ .

Fig 3-8



Ans:  $T = 13.3 \text{ kN}$

$F = 10.2 \text{ kN}$

**Exercise 3.7:**

The gusset plate is subjected to the forces of four members. Determine the force in member B and its proper orientation  $\theta$  for equilibrium. The forces are concurrent at point O. Take  $F = 12 \text{ kN}$ .

See Fig 3-8

Ans:  $T = 14.3 \text{ kN}$

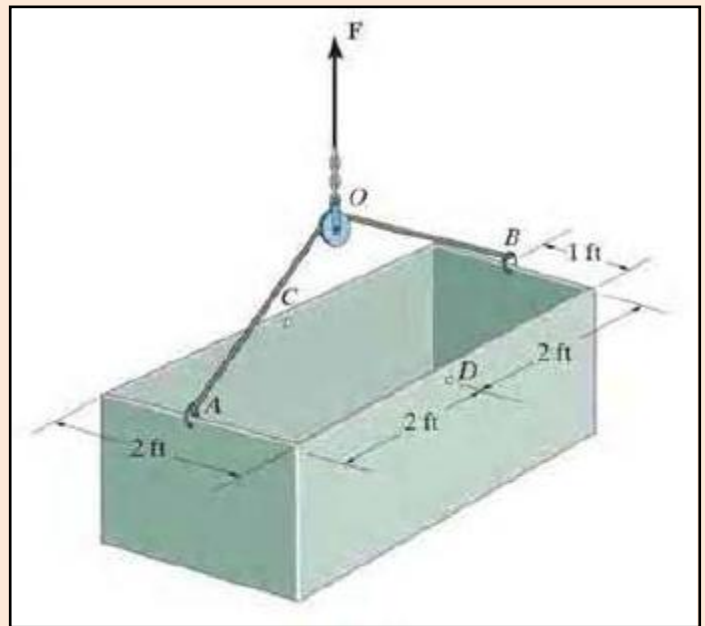
$\theta = 36.27^\circ$



**Exercise 3.8:**

The 200 lb uniform tank is suspended by means of a 6 ft long cable which is attached to the sides of the tank and passes over the small pulley located at O. If the cable can be attached at either points A and B or C and D. Determine which attachment produces the least amount of Tension in the cable. What is this tension?

Fig 3-9



Ans:  $T = 106 \text{ lb}$  related to CD attachment

### 3.4 Three dimensional force systems

In the case of three dimensional force system, as in fig 3.10, we can resolve the forces into their respective  $\vec{i}$ ,  $\vec{j}$ ,  $\vec{k}$  components **For equilibrium**, so that.

$$\sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_z \vec{k} = 0$$

بعد تحليل القوى إلى مركبات ثلاثية الأبعاد يمكن كتابة معادلة الاتزان الجسم في اتجاه كل محور.

To satisfy this equation we require

$$\sum F_x = 0$$

$$\sum F_y = 0$$

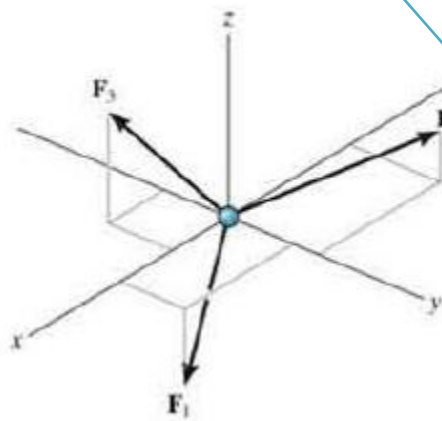
$$\sum F_z = 0$$

مجموع القوى في اتجاه المحور x تساوي 0.

مجموع القوى في اتجاه المحور y تساوي 0.

مجموع القوى في اتجاه المحور z تساوي 0.

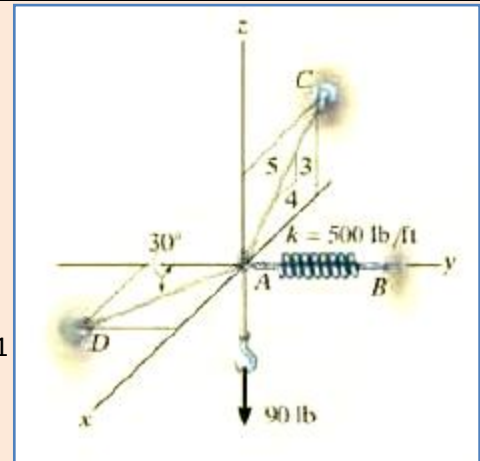
Fig 3-10



**Exercise 3.9:**

A 90 lb is suspended from the hook shown in fig 3-11. If the load is supported by two cables and a spring having a stiffness  $k = 500 \text{ lb/ft}$ , determine the force in cables and the stretch of the spring for equilibrium. Cable AD lies in the x-y plane and cable AC lies in x-z plane.

Fig 3-11

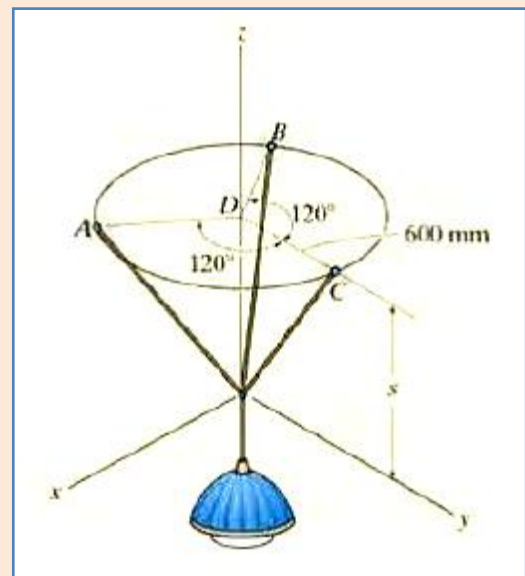


$$\text{Ans: } F_C = 150 \text{ lb} \quad F_D = 240 \text{ lb} \quad F_B = 207.8 \text{ lb} \quad S_{AB} = 0.416 \text{ ft}$$

**Exercise 3.10:**

The 10 kg lamp in fig 3.12 is suspended from the three equal length cords. Determine its smallest vertical distance  $s$  from the ceiling if the force developed in any cord is not allowed to exceed 50N.

Fig 3-12

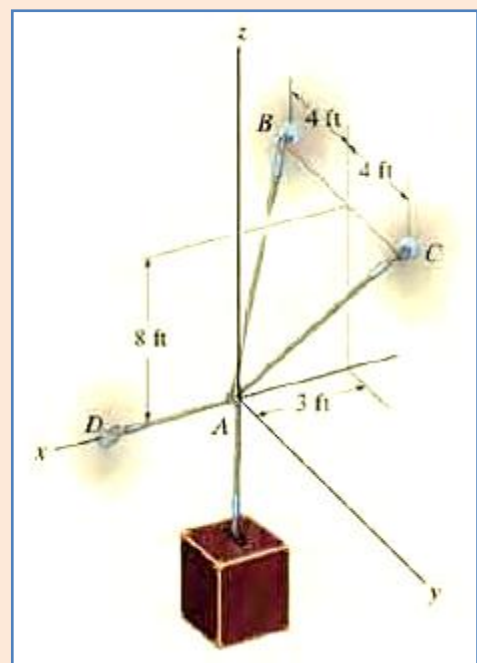


$$\text{Ans: } S = 519 \text{ mm}$$

**Exercise 3.11:**

Determine the force in each cable used to support the 40 lb crate shown fig 3-13.

Fig 3-13

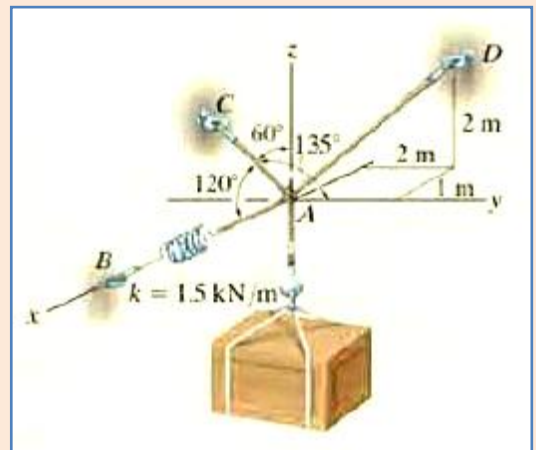


$$\text{Ans: } F_B = F_C = 23.6 \text{ lb} \quad F_D = 15 \text{ lb}$$

**Exercise 3.12:**

Determine the tension in each cord used to support the 100 kg crate shown fig 3-14.

Fig 3-14



$$\text{Ans: } F_C = 813 \text{ N}$$

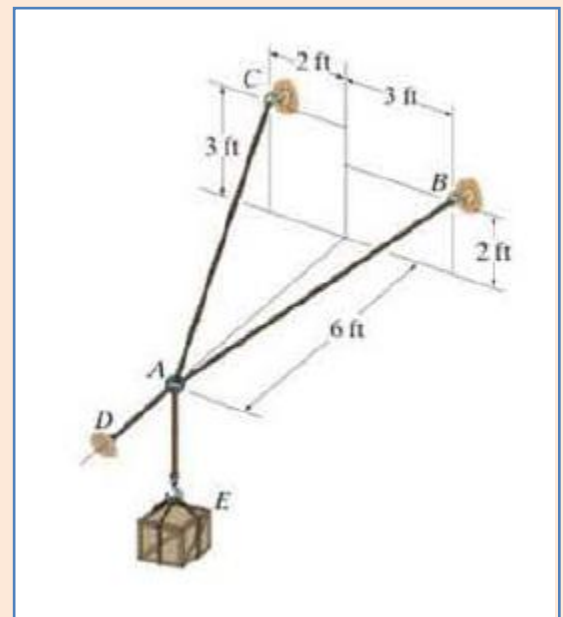
$$F_D = 862 \text{ N}$$

$$F_B = 694 \text{ N}$$

**Exercise 3.13:**

The 150 lb crate is supported by cables AB, AC and AD. Determine the tension in these wires.

Fig 3-15



$$\text{Ans: } F_B = 162 \text{ lb}$$

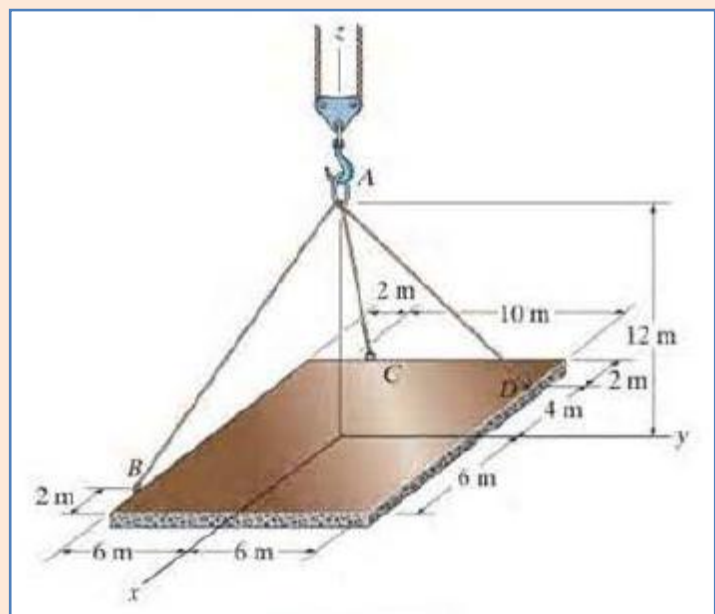
$$F_C = 242 \text{ lb}$$

$$F_D = 346 \text{ lb}$$

**Exercise 3.14:**

The ends of the three cables are attached to a ring at A and to the edge of a uniform 150 kg plate. Determine the tension in each of the cables for equilibrium.

Fig 3-16



$$\text{Ans: } F_B = 858 \text{ N}$$

$$F_C = 0 \text{ N}$$

$$F_D = 858 \text{ N}$$

# Chapter

Four

## Force System Resultantes

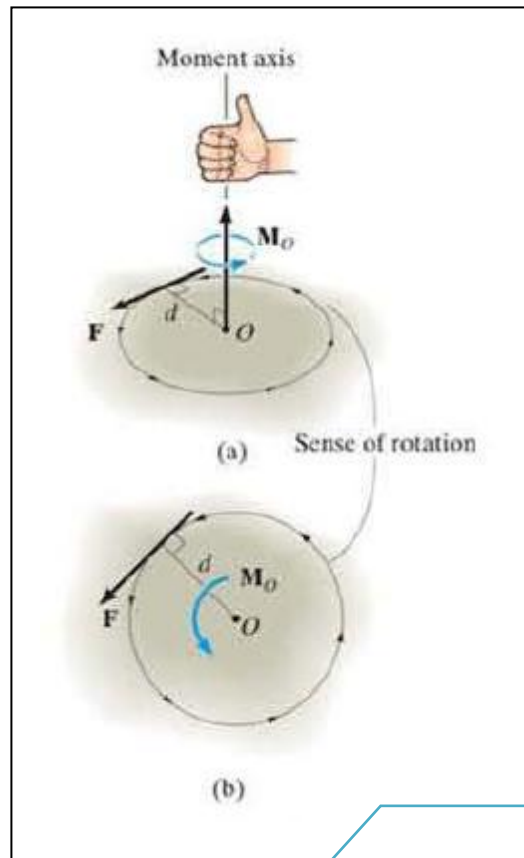
العلم صيد والكتابة قيده      قيد صيودك بالحبال الوثيقة  
فمن حماقة أن تصيد غزالة      وتتركها بين الخلائق طالقة

## Force System Resultants

### 4.1 Moment of a force scalar formulation.

The moment  $\vec{M}_O$  about point **O**, or about an axis passing through **O** and perpendicular to the plane, is a **vector quantity** since it has a specified **magnitude and direction** (fig 4-1).

Fig 4-1



العزم هو الأثر  
الدوراني الذي  
تحدثه القوة في  
الجسم.

فهو ممثل في متجه  
لقياس مدى قدرة  
قوة على تدوير  
جسم حول نقطة  
معينة أو محور ما.

تأثير الدوران يتناسب  
طرديا مع القوة والمسافة  
حول نقطة معينة ويعطى  
بحاصل ضرب مقدار  
القوة في ذراعها

The magnitude of  $M_O$  is

$$M_O = F \cdot d \quad 4.1$$

Where  $d$  is the **moment arm** or **perpendicular distance from the axis at point **O**** to the line of action of the force. **Units** of moment is **N.m** or **lb.ft**.

The **direction** of  $\vec{M}_O$  is defined by its moment axis which is **perpendicular** to the **plane** that contains the force **F** and its moment arm  $d$ . The **right-hand rule** is used establish the sense of the direction of  $\vec{M}_O$ .

يقصد بالذراع  $d$   
المسافة العمودية  
بين خط عمل القوة  
و النقطة المطلوب  
حساب العزم  
حولها.

For two dimensional problems, where all the forces lie within the x-y plane, fig 4-2, the resultant moment  $(\mathbf{M}_R)_O$  about point O (the z axis) can be determined by finding the **algebraic sum of the moments** caused by all the forces in the system. As a **convention** we will generally consider **positive moments** as a **counterclockwise** since they are directed along the positive z axis (out of page). **Clockwise** moments will be **negative**. Using the sign convention, the resultant moment in fig 4-3 is therefore

$$(M_R)_O = \sum Fd$$

$$(M_R)_O = F_1d_1 - F_2d_2 + F_3d_3$$

إذا كان لدينا جسم تحت تأثير مجموعة من القوى، فإن قيمة العزم حول نقطة معينة تساوي الجمع الجبري لعزوم القوى منفردة

اتفاق:

يعتبر العزم موجب إذا كان اتجاهه عكس اتجاه عقارب الساعة و سالب إذا كان اتجاهه هو نفس اتجاه عقارب الساعة

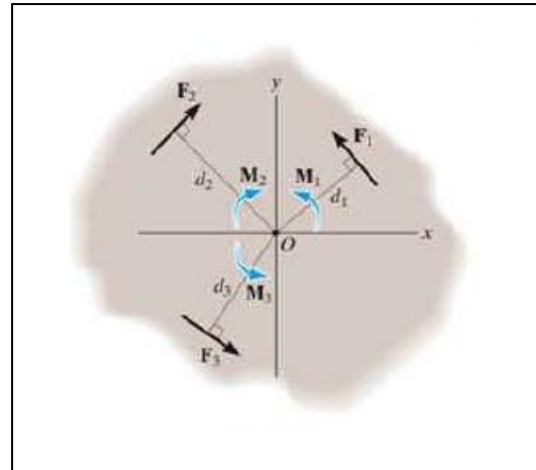
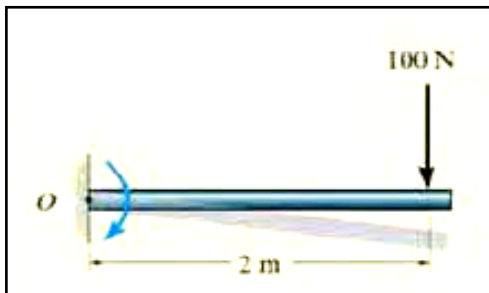


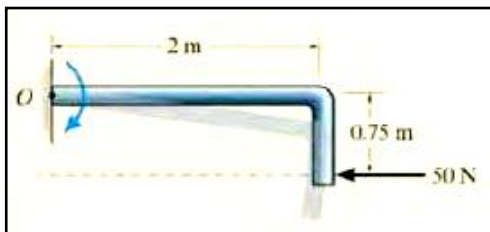
Fig 4-2

### Example:

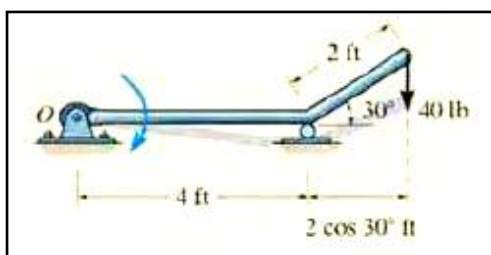
For each case illustrated below, the moments of the forces are:



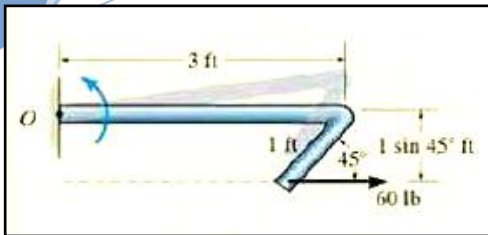
$$M_O = (100 \text{ N})(2 \text{ m}) = 200 \text{ N} \cdot \text{m} \curvearrowright$$



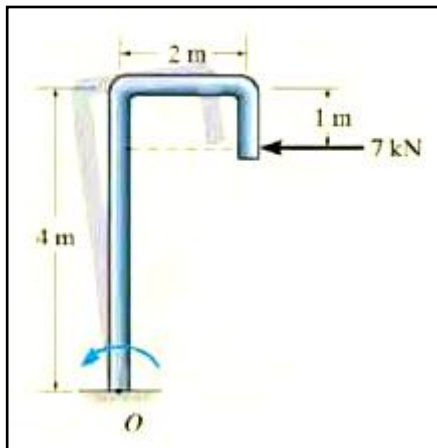
$$M_O = (50 \text{ N})(0.75 \text{ m}) = 37.5 \text{ N} \cdot \text{m} \curvearrowright$$



$$M_O = (40 \text{ lb})(4 \text{ ft} + 2 \cos 30^\circ \text{ ft}) = 229 \text{ lb} \cdot \text{ft} \curvearrowright$$



$$M_O = (60 \text{ lb})(1 \sin 45^\circ \text{ ft}) = 42.4 \text{ lb} \cdot \text{ft} \curvearrowleft$$



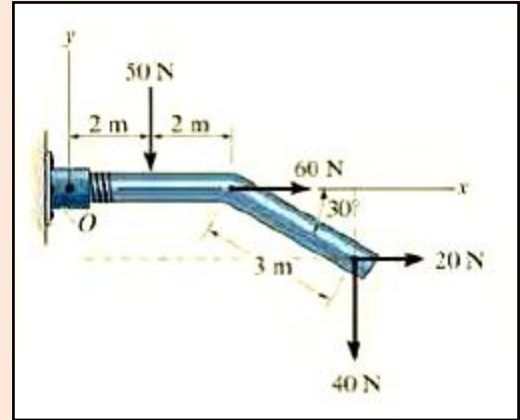
$$M_O = (7 \text{ kN})(4 \text{ m} - 1 \text{ m}) = 21.0 \text{ kN} \cdot \text{m} \curvearrowleft$$



**Exercise 4.1:**

Determine the resultant moment of the four forces acting on the rod shown in fig 4-3 about point O.

Fig 4-3



Ans:  $M_{R_0} = -334 \text{ N.m}$

## 4.2 Cross product

The cross product of two vectors  $\vec{A}$  and  $\vec{B}$  yields the vector  $\vec{C}$  which is written

$$\vec{C} = \vec{A} \times \vec{B} \quad 4.2$$

And read  $\vec{C}$  equals  $\vec{A}$  cross  $\vec{B}$ .

The magnitude of  $\vec{C}$  is defined as the product of the magnitudes  $\vec{A}$  and  $\vec{B}$  and the sine of the angle  $\theta$  between their tails ( $0^\circ \leq \theta \leq 180^\circ$ ), thus

$$C = A B \sin \theta$$

$\vec{C}$  has a direction that is perpendicular to the plane containing  $\vec{A}$  and  $\vec{B}$  such that  $\vec{C}$  is specified by the right-hand rule.

Knowing both the magnitude and direction of  $\vec{C}$ , we can write

$$\vec{C} = \vec{A} \times \vec{B} = (A B \sin \theta) \vec{u}_C \quad 4.3$$

Where the scalar  $(A B \sin \theta)$  defines the magnitude of  $\vec{C}$  and the unit vector  $\vec{u}_C$  defines the direction of  $\vec{C}$  (fig 4-4).

Laws of operation:

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

الضرب الإتجاهي: سمي بهذا الاسم لأن ناتج الضرب عبارة عن كمية اتجاهية (vector)، بمعنى، أن حاصل الضرب الإتجاهي لمتجهين هو متجه ثالث، اتجاهه يكون عموديا على المستوي الذي يحوي المتجهين المضروبين ببعضهما.

مقدار المتجه الثالث يعطى بعلاقة  $C$ .

يمكن معرفة اتجاه المتجه الثالث باستخدام قاعدة اليد اليمنى.

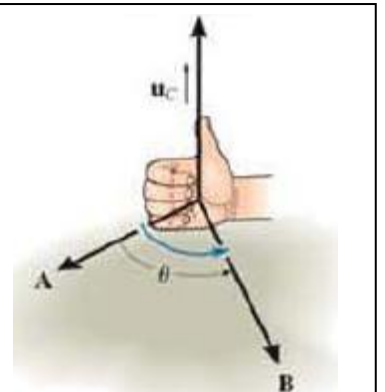


Fig 4-4



$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \quad (\text{commutative law is not valid})$$

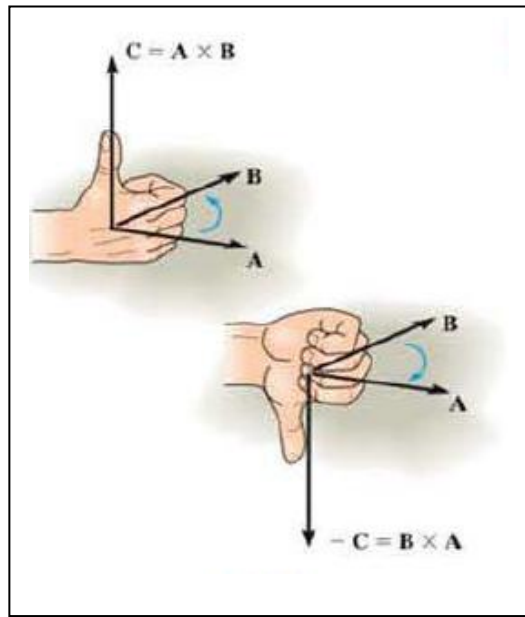


Fig 4-5

$$a(\vec{A} \times \vec{B}) = (a\vec{A}) \times \vec{B} = \vec{A} \times (a\vec{B}) = (\vec{A} \times \vec{B})a \quad (\text{associative law})$$

عملية الضرب الإتجاهي  
تجميعية

$$\vec{A} \times (\vec{B} + \vec{D}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{D}) \quad (\text{distributive law})$$

عملية الضرب الإتجاهي  
توزيعية

### Cartesian vector formulation:

Equation 4.3 may be used to find the cross product of any pair of Cartesian unit vectors. For example, to find  $\vec{i} \times \vec{j}$ , the magnitude of the resultant vector is

$$(i)(j)\sin 90^\circ = (1)(1)(1) = 1$$

$$(i)(i)\sin 0^\circ = 0$$

and its direction is determined using the right-hand rule (fig 4-6), the resultant vector points in the  $+\vec{k}$  direction. Thus  $\vec{i} \times \vec{j} = (1)\vec{k}$ .

In similar maner.

$$\begin{array}{lll} \vec{i} \times \vec{j} = \vec{k} & \vec{i} \times \vec{k} = -\vec{j} & \vec{i} \times \vec{i} = 0 \\ \vec{j} \times \vec{k} = \vec{i} & \vec{j} \times \vec{i} = -\vec{k} & \vec{j} \times \vec{j} = 0 \\ \vec{k} \times \vec{i} = \vec{j} & \vec{k} \times \vec{j} = -\vec{i} & \vec{k} \times \vec{k} = 0 \end{array}$$

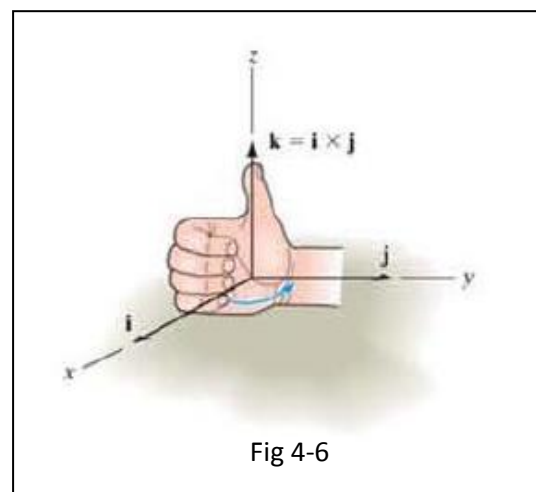


Fig 4-6

للتبسيط نستخدم الدائرة التالية  
من أجل إيجاد نتائج الضرب  
الاتجاهي لمتجهات الوحدة.

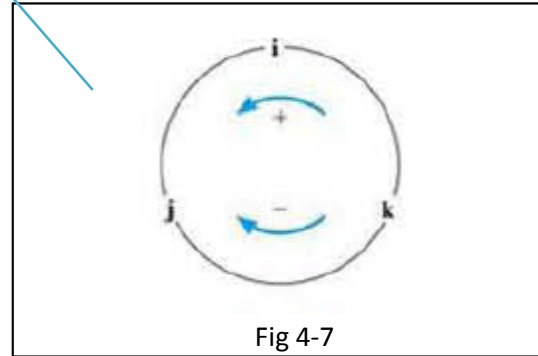


Fig 4-7

A simple scheme shown in fig 4-7 is helpful for obtaining the same results when the need arises.

Let us now consider the cross product of two general vectors  $\vec{A}$  and  $\vec{B}$ .

$$\vec{A} \times \vec{B} = (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) \times (B_x \vec{i} + B_y \vec{j} + B_z \vec{k})$$

$$\begin{aligned} \vec{A} \times \vec{B} &= (A_x B_x (\vec{i} \times \vec{i}) + A_x B_y (\vec{i} \times \vec{j}) + A_x B_z (\vec{i} \times \vec{k}) \\ &\quad + A_y B_x (\vec{j} \times \vec{i}) + A_y B_y (\vec{j} \times \vec{j}) + A_y B_z (\vec{j} \times \vec{k}) \\ &\quad + A_z B_x (\vec{k} \times \vec{i}) + A_z B_y (\vec{k} \times \vec{j}) + A_z B_z (\vec{k} \times \vec{k})) \end{aligned}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \vec{i} - (A_x B_z - A_z B_x) \vec{j} + (A_x B_y - A_y B_x) \vec{k}$$

This equation may also be written in a more compact determinant form as

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

الكتابة الديكارتية للضرب  
الاتجاهي تعطى بالمحددات كما  
هو مبين في العلاقة التالية.

### 4.3 Moment of a force – vector formulation

The moment of a force  $F$  about a point  $O$  (fig 4-8) can be expressed using the vector cross product namely

$$\vec{M}_O = \vec{r} \times \vec{F} \quad 4.4$$

Here  $\vec{r}$  represents a position vector direct from  $O$  to any point on the line of action of  $\vec{F}$ .

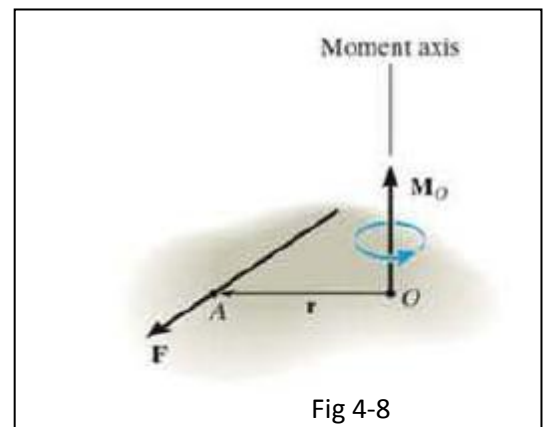


Fig 4-8

The magnitude of the cross product is defined from Eq. 4-3 as

$$M_0 = r F \sin \theta$$

الصيغة الاتجاهية للضرب الاتجاهي تعطى  
بالعلاقة التالية

where  $\theta$  is measured between the tails of  $\vec{r}$  and  $\vec{F}$ .

The direction and sense of  $\vec{M}_O$  in Eq. 4-4 are determined by the right-hand rule as it applies to the cross product (fig 4-9).

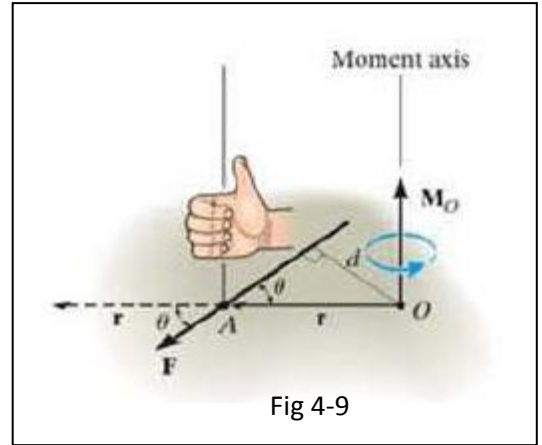


Fig 4-9

### Cartesian vector formulation:

If we establish  $x, y, z$  coordinate axes, then the position vector  $\vec{r}$  and force  $\vec{F}$  can be expressed as Cartesian vectors (fig 4-10)

$$\vec{M}_O = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Where  $r_x, r_y, r_z$  represent the  $x, y, z$  components of the position vector drawn from point  $O$  to any point on the line of action of the force.

$F_x, F_y, F_z$  represent the  $x, y, z$  of the force vector.

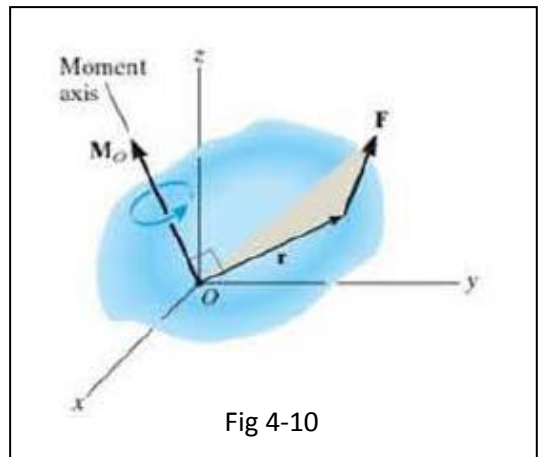


Fig 4-10

### Resultant Moment of a system of forces:

If a body is acted upon by a system of forces (fig 4-11), the resultant moment of the forces about point  $O$  can be determined by vector addition of the moment of each force. This resultant can be written symbolically as

$$\vec{M}_{R_O} = \sum (\vec{r} \times \vec{F})$$

إذا كان لدينا جسم تحت تأثير مجموعة من القوى، فإن قيمة العزم حول نقطة معينة تساوي جمع عزوم القوى المنفردة.

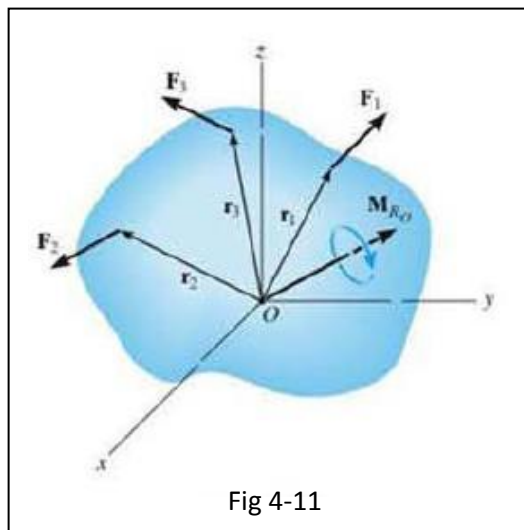
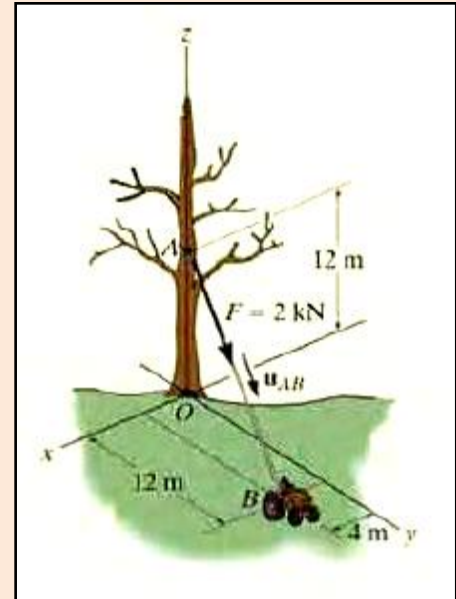


Fig 4-11

**Exercise 4.2:**

Express the moment produced by the force  $\vec{F}$  in Fig 4-12 about point O, as a Cartesian vector.

Fig 4-12

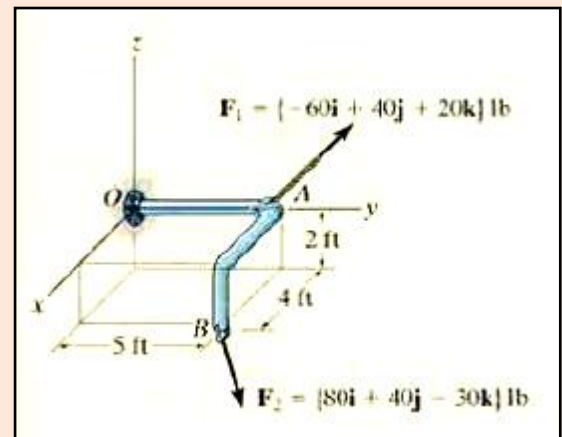


$$\text{Ans: } \vec{M}_{R_0} = \sum (\vec{r} \times \vec{F}) = \{-16.5\mathbf{i} - 5.51\mathbf{j}\} \text{ kN}\cdot\text{m}$$

**Exercise 4.3:**

Two forces act on the rod shown in fig 4-13. Determine the resultant moment they create about the flange at O. Express the result as a Cartesian vector.

Fig 4-13



$$\text{Ans: } \vec{M}_{R_0} = \sum (\vec{r} \times \vec{F}) = \{30\mathbf{i} - 40\mathbf{j} + 60\mathbf{k}\} \text{ lb}\cdot\text{ft}$$

## 4.4 Principle of moments

The principle of moments is referred to the French mathematician Varignon (1654-1722). It states that the moment of a force about a point is equal to the sum of the moments of the components of the force about the point. If we consider the case of fig 4-14, we have.

تنص نظرية فارنيون أن  
عزم قوة حول نقطة يساوي  
مجموع عزوم مركبات هذه  
القوة حول نفس النقطة.

$$\vec{M}_O = \vec{r} \times \vec{F} = \vec{r} \times (\vec{F}_1 + \vec{F}_2) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2$$

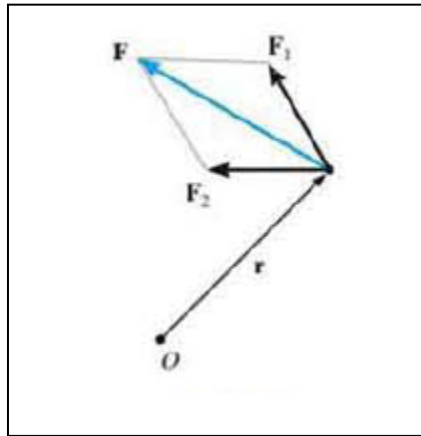


Fig 4-14

**Exercise 4.4:**

Determine the moment of the force in fig 4-15 about the point  $O$ .

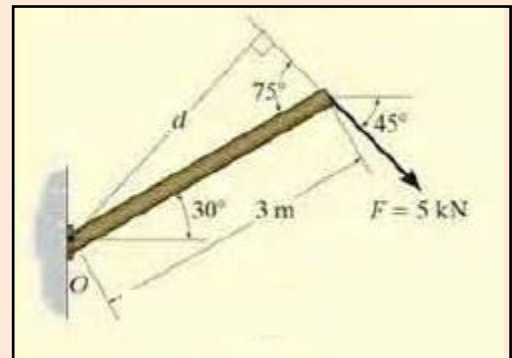


Fig 4-15

Ans:  $M_{R_0} = -14.5 \text{ kN.m}$

**Exercise 4.5:**

Determine the moment of the force in fig 4-16 about point  $O$ . Express the result as a Cartesian vector.

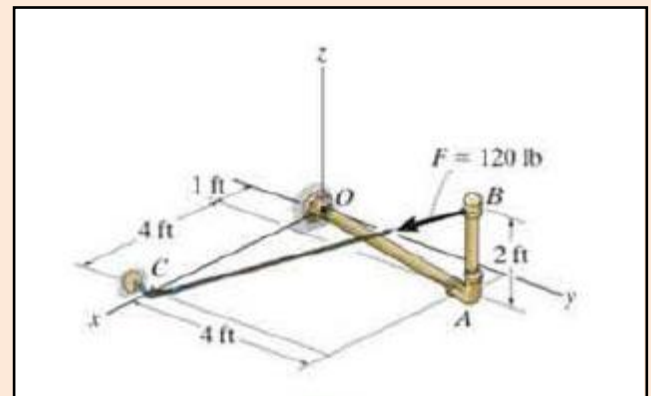


Fig 4-16

Ans:  $\vec{M}_O = \{200\vec{j} - 400\vec{k}\} \text{ lb.ft}$

**Exercise 4.6:**

Force  $\vec{F}$  acts at the end of the angle bracket shown in fig 4-17. Determine the moment of the force about point  $O$ .

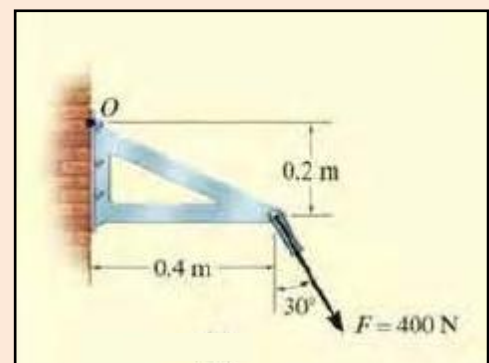


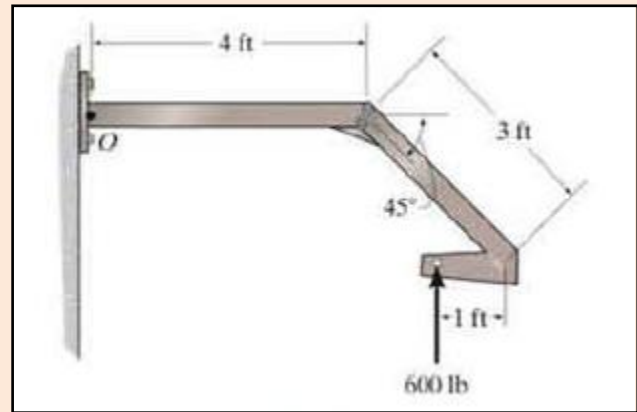
Fig 4-17

Ans:  $\vec{M}_O = \{-98.6\vec{k}\} \text{ N.m}$

**Exercise 4.7:**

Determine the moment of the force about point O.

Fig 4-18

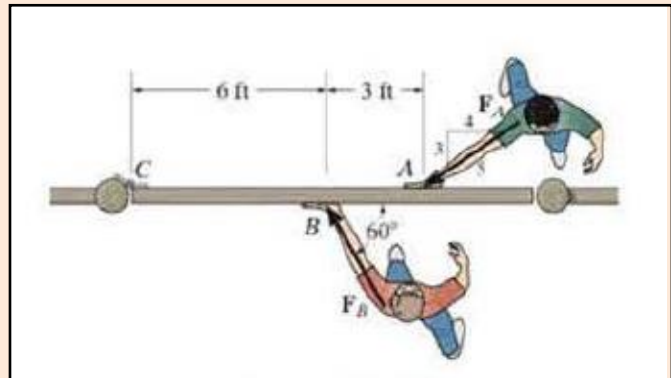


Ans:  $M_0 = 36.7 \text{ N.m}$

**Exercise 4.8:**

The two boys push the gate with forces of  $F_A = 30 \text{ lb}$  and  $F_B = 50 \text{ lb}$  as shown. Determine the moment of each force about C. Which way will the gate rotate clockwise or counterclockwise? Neglect the thickness of the gate.

Fig 4-19



Ans:  $(M_{F_A})_C = -162 \text{ lb.ft}$        $(M_{F_B})_C = 260 \text{ lb.ft}$   
 since  $(M_{F_B})_C > (M_{F_A})_C$  the gate will rotate counterclockwise

**Exercise 4.9:**

Two boys push on the gate as shown. If the boy at B exerts a force of  $F_B = 30 \text{ lb}$ , determine the magnitude of the force  $F_A$  the boy at A must exert in order to prevent the gate from turning. Neglect the thickness of the gate.

See Fig 4-19

Ans:  $F_A = 28.9 \text{ lb}$

## 4.5 Moment of a Force about a specified axis

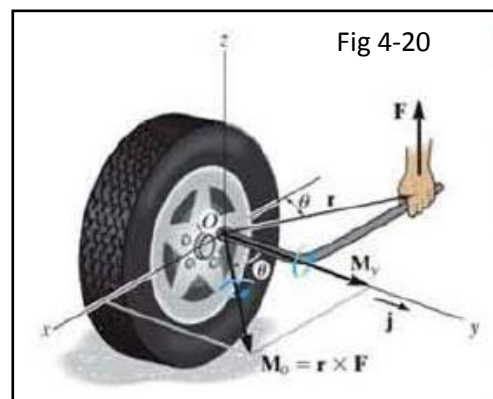
### Scalar analysis

In general, for any axis (fig 4-20) the moment is

$$M_a = F d_a$$

### Vector Analysis

If the vectors are written in Cartesian form, we have



$$M_a = \vec{u}_a \cdot (\vec{r} \times \vec{F}) = \begin{vmatrix} u_{ax} & u_{ay} & u_{az} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Where  $u_{ax}, u_{ay}, u_{az}$  represent the  $x, y, z$  components of unit vector defining the direction of the  $a$  axis.

$r_x, r_y, r_z$  represent the  $x, y, z$  components of the position vector extended from any point  $O$  on the  $a$  axis to any point  $A$  on the line of action of the force.

$F_x, F_y, F_z$  represent the  $x, y, z$  components of the force vector.

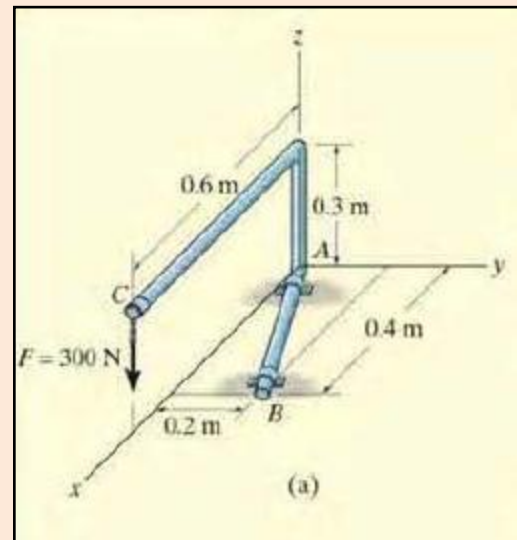
Once  $M_a$  is determined, we can then express  $\vec{M}_a$  as a Cartesian vector namely.

$$\vec{M}_a = M_a \vec{u}_a$$

**Exercise 4.10:**

Determine the moment  $M_{AB}$  produced by the force  $\vec{F}$  in fig 4-21, which tends to rotate the rod about the AB axis.

Fig 4-21

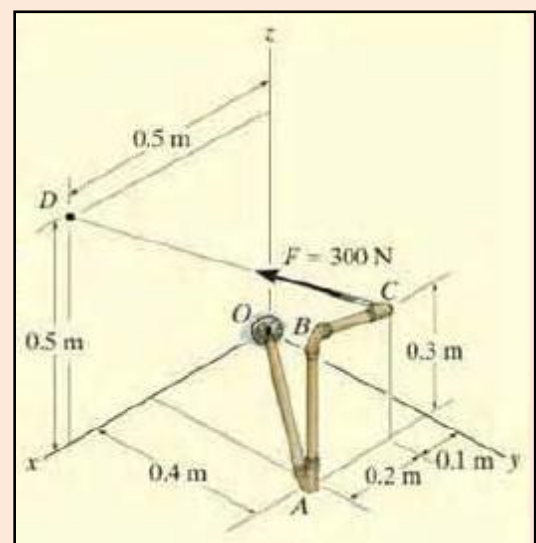


Ans:  $\vec{M}_{AB} = \{72\vec{i} + 36\vec{j}\}$  N.m

**Exercise 4.11:**

Determine the magnitude of the moment of force  $\vec{F}$  about segment OA of force the pipe assembly in fig 4-24a

Fig 4-22

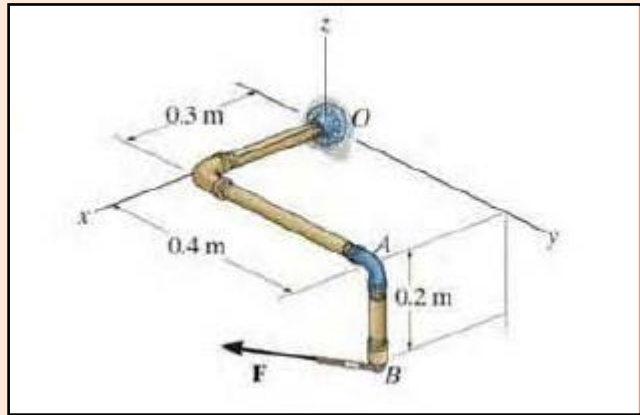


Ans:  $M_{OA} = 100$  N.m

**Exercise 4.12:**

Determine the magnitude of the moment of the force  $\vec{F} = \{300\vec{i} - 200\vec{j} + 150\vec{k}\}$  N about the x axis. Express the result as a Cartesian vector.

Fig 4-23



Ans:  $M_x = 20 \text{ N.m}$

**Exercise 4.13**

Determine the magnitude of the moment of the force  $\vec{F} = \{300\vec{i} - 200\vec{j} + 150\vec{k}\}$  N about the OA axis. Express the result as a Cartesian vector

See Fig 4-23

Ans:  $M_{OA} = -72 \text{ N.m}$

## 4.6 Moment of a couple

a couple is defined as a two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance  $d$  (fig 4-24). The moment produced by a couple is called a couple moment.

### Scalar Formulation

The moment of a couple  $\vec{M}$  (fig 4-25), is defined as having a magnitude of

$$M = F d$$

Where  $F$  is the magnitude of one of the forces and  $d$  is the perpendicular distance or moment arm between the forces. The direction and sense of the couple moment are determined by the right hand rule.  $\vec{M}$  will act perpendicular to the plane containing these forces.

يعرف عزم الازدواج بأنه تأثير قوتين متساويتين في المقدار و متوازيتين و متضادتين في الاتجاه يثران في نقطة معينة.

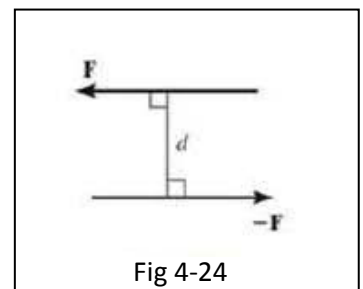


Fig 4-24

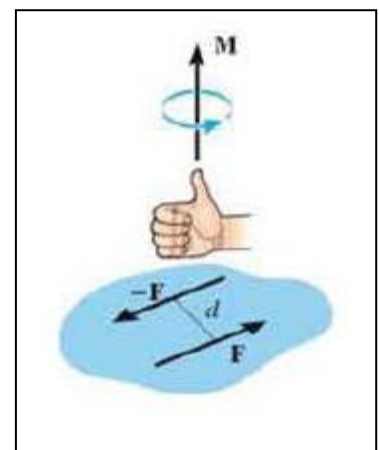


Fig 4-25



**Vector Formulation**

The moment of a couple can also be expressed by the vector Cross product as

$$\vec{M} = \vec{r} \times \vec{F}$$

**Resultant couple moment**

Since couple moments are vectors, their resultant can be determined by vector addition.

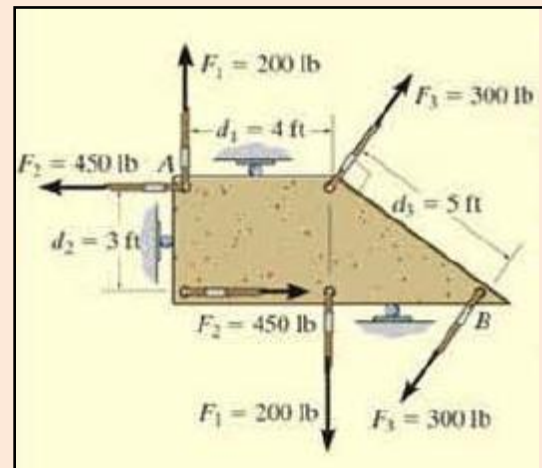
If more than two couple moments act on the body, we may generalize this concept and write the vector resultant as

$$\vec{M}_R = \sum (\vec{r} \times \vec{F})$$

**Exercise 4.14:**

Determine the resultant couple moment of the three couples acting on the plate in fig 4-26.

Fig 4-26

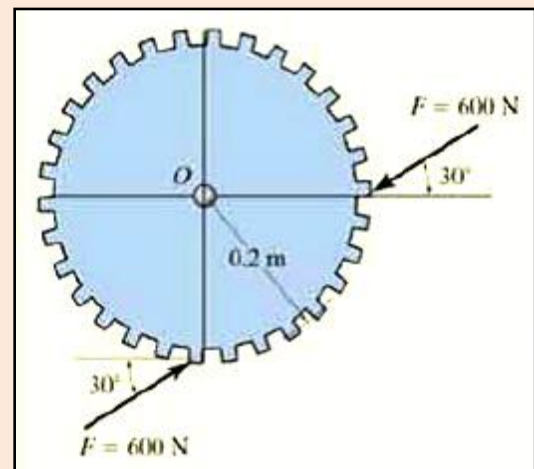


Ans:  $M_R = -950 \text{ lb}\cdot\text{ft}$

**Exercise 4.15:**

Determine the magnitude and direction of the couple moment acting on the gear in fig 4-27.

Fig 4-27

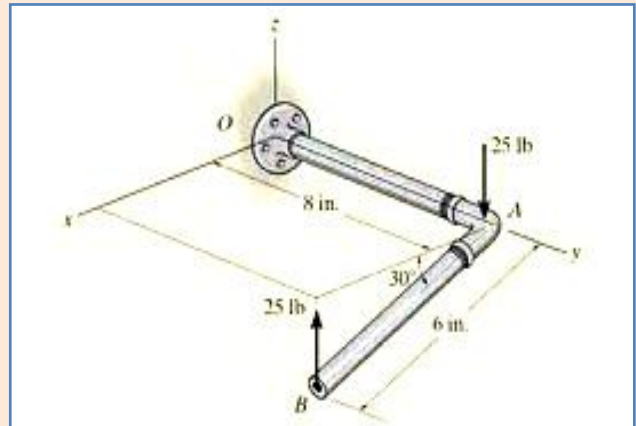


Ans:  $M = 43.9 \text{ N}\cdot\text{m}$

**Exercise 4.16:**

Determine the couple moment acting on the pipe shown in fig 4-28 Segment AB is directed  $30^\circ$  below the x-y plane. Take  $OA=8$  in and  $AB=6$  in.

Fig 4-28

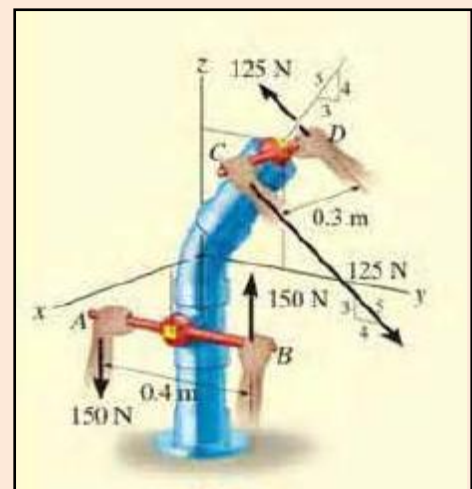


Ans:  $M = -130 \text{ lb.in}$

**Exercise 4.17:**

Replace the two couples acting on the pipe Column in fig 4-29 by a resultant couple moment.

Fig 4-29

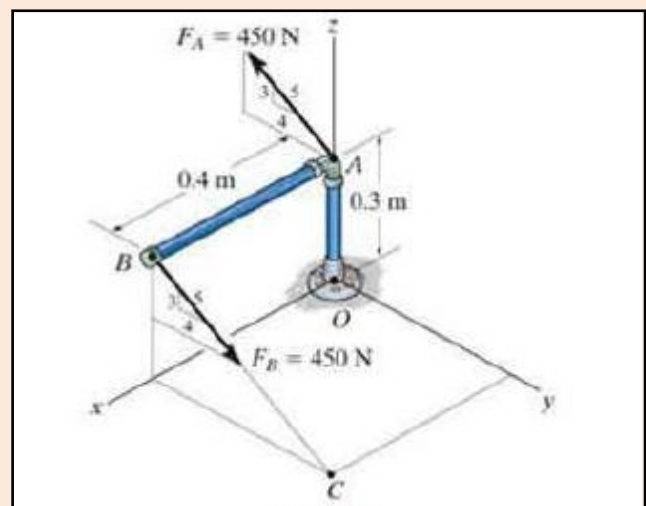


Ans:  $\vec{M}_R = \{60 \vec{i} + 22.5 \vec{j} + 30 \vec{k}\} \text{ N.m}$

**Exercise 4.18:**

Determine the couple moment acting on the pipe assembly and express the result as a Cartesian vector.

Fig 4-30



Ans:  $\vec{M}_C = \vec{r}_{AB} \times \vec{F}_B = \{108 \vec{j} + 144 \vec{k}\} \text{ N.m}$

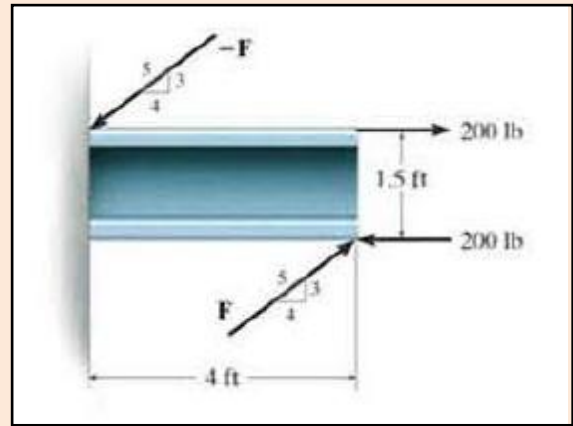
**Exercise 4.19:**

Two couples act on the beam as shown.

Determine the magnitude of  $\vec{F}$  so that the resultant couple moment is 300 lb.ft couterclockwise.

Where on the beam does the resultant couple act?

Fig 4-31



Ans:  $F = 167$  lb

# Chapter

Five

## Equilibrium Of a Rigid Body

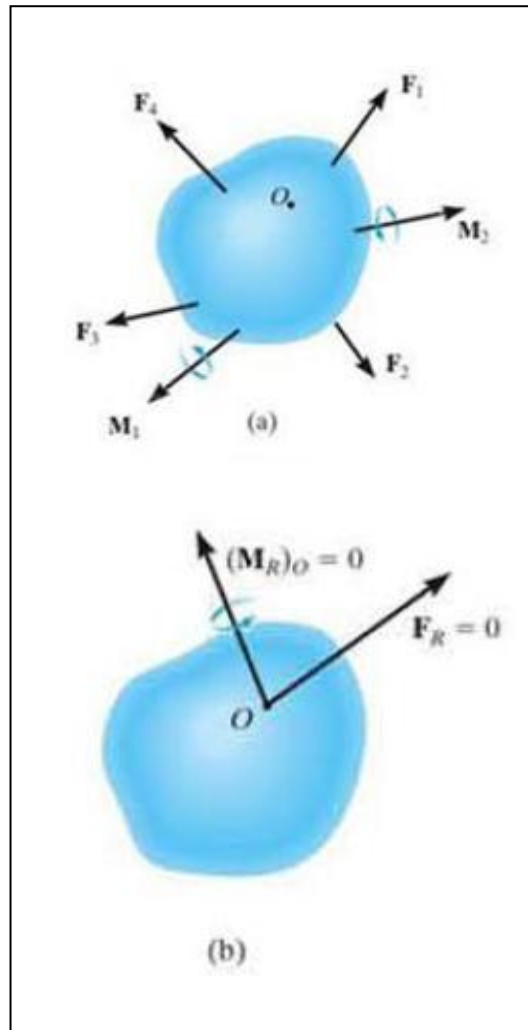
شكوت إلى وبيع سوء حفظي      فأرشدني إلى ترك المعاصي

## Equilibrium Of A Rigid Body

### 5.1 Conditions for Rigid-Body Equilibrium

As shown in fig 5-1 the body is subjected to an external force couple moment system that is the result of the effects of **gravitational**, **Electrical**, **magnetic**, or contact forces caused by adjacent bodies.

Fig. 5-1



عندما يكون أي جسم تحت تأثير قوة ما فإنه سيحاول التحرك في اتجاه تأثير القوة وبالتالي يصبح الجسم في حالة عدم اتزان، كذلك عندما يكون الجسم تحت تأثير مجموعة من القوى و مجموعة من العزوم فإن الجسم سوف يتحرك تحت تأثير القوى و العزوم، والحالة الوحيدة التي يكون فيها الجسم في حالة اتزان هو عندما تكون محصلة القوى الخارجية و كذلك العزوم تساوي صفراً.

Using the methods of the previous chapter, the force and couple moment system acting on a body can be reduced to an equivalent resultant force and resultant couple moment at any arbitrary point O on or off the body, Fig 5-1 b. **If this resultant force and couple moment are both equal to zero then the body is said to be in equilibrium.**

Mathematically. The equilibrium of a body is expressed as

$$\vec{F}_R = \sum \vec{F} = 0$$

$$(\vec{M}_R)_O = \sum \vec{M}_O = 0$$

**5.2 Free-Body Diagrams**

This **diagram** is a **sketch of the outlined shape of the body**, which represents is as being **isolated or "free"** from its surroundings, i.e. a "free body" on this sketch it is necessary to **show all the forces and couple moments that the surroundings exert on the body** so that these **effects** can be **accounted for when the equations of equilibrium are applied**. A thorough understanding of how to draw a free-body diagram is of primary importance for solving problems in mechanics.

**Support Reactions:** Before presenting a formal procedure as to how to draw a free body diagram, we will first **consider the various types of reactions that occur at supports and points of contact between bodies** subjected to coplanar force systems. As a general rule.

- If a **support prevents the translation of a body in a given direction**, then a **force is developed on the body in that direction**.
- If **rotation is prevented**, a **couple moment is exerted on the body**.

For **example** let us consider **three ways** in which a horizontal member. Such as a **beam** is supported at its end. One method consists of a roller or cylinder, Fig.5-2a. Since this support only prevents the beam from **translating** in the vertical direction, the roller will only exert a force on the beam in this direction, Fig.5-2b.

The beam can be supported in a more restrictive manner by using a pin, fig. 5-2c. The pin passes through a hole in the beam and two leaves which are fixed to the ground. Here the pin can prevent translation of the beam in any direction  $\emptyset$ , Fig.5-2d, and so the pin must exert a force  $F$  on the beam this direction. For purposes of analysis, it is generally easier to represent this resultant force  $\vec{F}$  by its two rectangular components  $F_x$  and  $F_y$ , fig 5-2e. If  $F_x$  and  $F_y$  are known then  $F$  and  $\emptyset$  can be calculated.

The most restrictive way to support the beam would be to use a fixed support as shown in fig 5-2f. This support will prevent both translation and rotation of the beam. To do this a force and couple moment must be developed on the beam at its point of connection. Fig 5-2g. As in the case of the pin, the force is usually represented by its rectangular components  $F_x$  and  $F_y$ .

لإيجاد معادلات الاتزان  
لجسم ما لابد من عزل  
الجسم من المحيط الذي  
هو فيه واستبدالها بردود  
أفعال و قوى خارجية في  
رسم يسمى بالرسم البياني  
للجسم الحر. هذا الرسم  
يوضح فيه الجسم الأصلي  
مع القوى الخارجية و  
العزوم وردود الأفعال  
(مثل الدعامات). الرسم  
البياني للجسم الحر له  
أهمية قصوى لدراسة  
وحل المسائل المتعلقة  
بالميكانيكا و الإستاتيكا.

قواعد عامة حول ردود أفعال عند نقاط  
التلامس الأجسام و الدعامات:

- إذا منع الجسم من الحركة  
في اتجاه معين فسيتم عرض  
إلى قوة مضادة و في نفس  
اتجاه حركة ذلك الجسم.
- إذا منع جسم ما من الدوران  
فسيكون ذلك الجسم عرضة  
لعزم ازدواج.

المثال التالي هو حول ردود الأفعال لبعض  
الدعامات:

- الحالة الأولى: دعامة منزلقة (roller support)  
ولها رد فعل عمودي.
- الحالة الثانية: دعامة المفصلية  
(hinge support) ولها رد فعل  
أفقي و عمودي وليس لها عزم.
- الحالة الثالثة: دعامة ثابتة (كابولية)  
(fixed or cantiliver support)  
ولها رد فعل عمودي و أفقي و عزم.

الهدف الأساسي للدعامات (الركائز) هو تحمل الكمرات وتحويل الأحمال في المبنى إلى عناصر أخرى مثل عمود أو حائط أو الأساس.

- الدعامة المنزلقة (Roller support): هذا النوع من الدعامات يسمح للكمرة أن تتحرك في اتجاه محور الكمرة، مع العلم بأن الكمرة يمكن أن تتعرض لإجهاد داخلي ناتج عن التمدد بسبب تغير الحرارة الخارجية وبالتالي فإن الدعامة المنزلقة تسمح للكمرة بأن تتمدد في نطاق معين.
- الدعامة المفصليّة (Hinge support): الدعامة المفصليّة لا تسمح بالتحرك في الاتجاه الأفقي ولا في الاتجاه العمودي وتسمح بالدوران حول نقطة الالتماس.
- الدعامة الثابتة أو الكابولية (fixed or cantiliver support): الدعامة الثابتة لا تسمح بالحركة في الاتجاه الأفقي و العمودي ولا الدوران حول نقطة التثبيت.

Fig 5-2

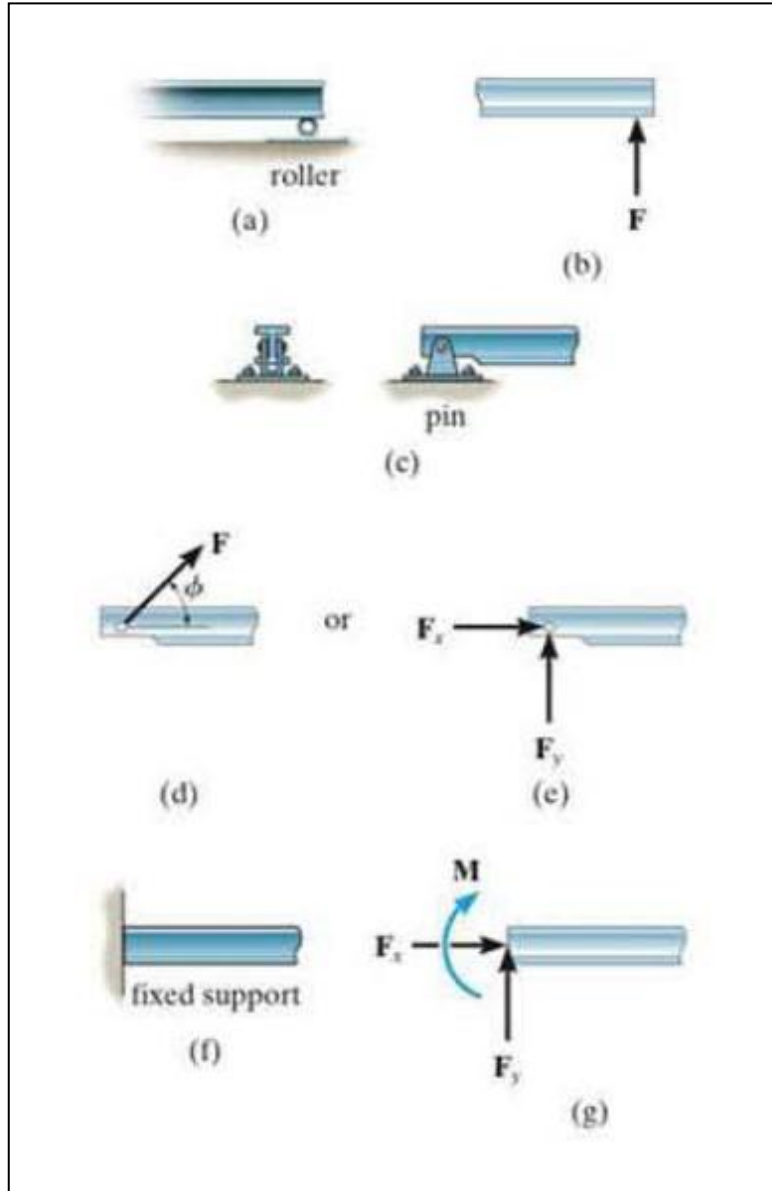

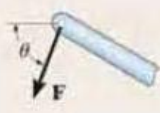

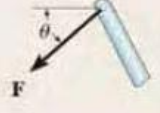









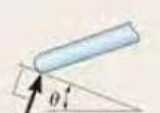







Table 5-1 lists other **common types of supports** for bodies subjected to coplanar force systems. (In all cases the angle  $\theta$  is assumed to be known.)






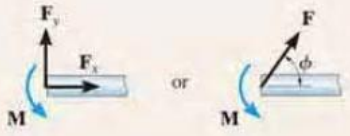
الجدول التالي يعطي قائمة  
من الدعامات مع ردود  
الأفعال عند نقاط الالتماس.

**TABLE 5-1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems**

Types of Connection	Reaction	Number of Unknowns
1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
4)  roller or pin in inclined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
6)  tooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
7)  member pin connected collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

*continued*



TABLE 5-1 Continued		
Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction $\phi$ of the resultant force. Note that $\phi$ and $\theta$ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction $\phi$ of the resultant force.

To **construct a free-body diagram** for a rigid body or any group of bodies considered as a single system. The following steps should be performed:

**Draw outlined shape:** Imagine the body to be isolated or cut "free" from its constraints and connections and draw (sketch) its outlined shape.

### Show all Forces and Couple moments:

Identify all the known and unknown external forces and couple moments act on the body. Those generally encountered are due to (1) applied loadings (2) reactions occurring at the supports or at points of contact with other bodies (see table 5-1) and (3) the weight of the body. To account for all these effects, it may help to trace over the boundary, carefully noting each force or couple moment acting on it.

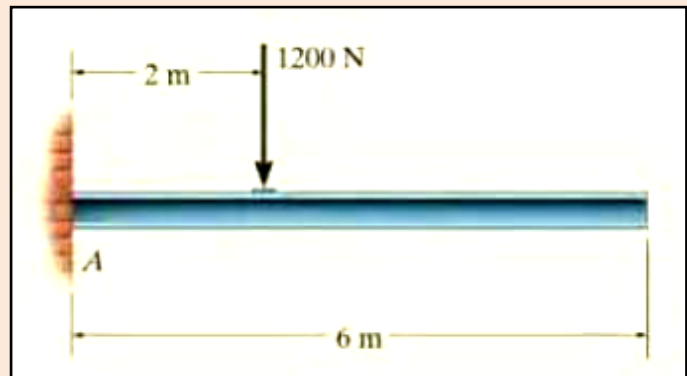
### Identify each loading and give dimensions:

The forces and couple moments that are known should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and direction angles of forces and couple moments that are unknown. Establish an  $x, y$  coordinate system so that these unknowns,  $A_x, A_y$ , etc can be identified. Finally indicate the dimensions of the body necessary for calculating the moments of forces.

**Exercise 5.1:**

Draw the free-body diagram of the uniform beam shown in Fig. 5-3. The beam has a mass of 100 kg.

Fig 5-3

**Exercise 5.2:**

Draw the free-body diagram of the foot lever shown in fig.5-4. The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in, and the force in the short link at B is 20 lb.

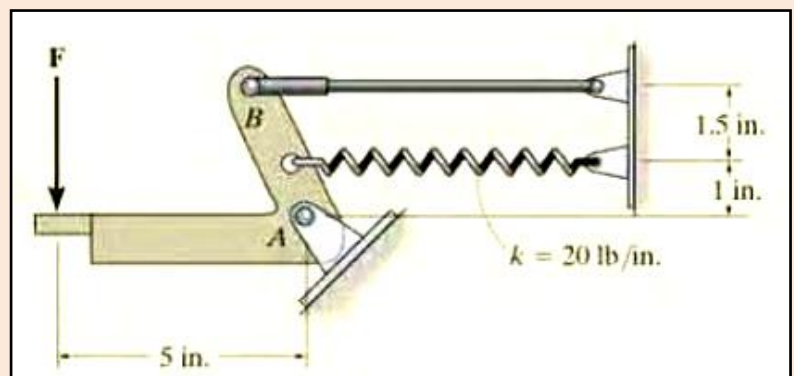


Fig 5-4

**Exercise 5.3:**

Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor in fig 5-5. Draw the free-body diagrams for each pipe and both pipes together.

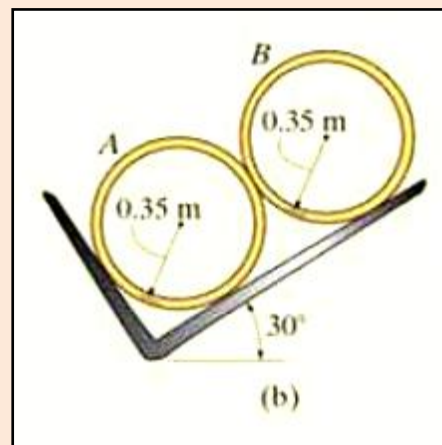


Fig 5-5

**Exercise 5.4:**

Draw the free-body diagram of the unloaded platform that is suspended off the edge of the oil rig shown in fig 5-6.

The platform has a mass of 200 kg.



Fig 5-6

### 5.3 Equations of equilibrium

In sec. 5.1 we developed the two equations which are both **necessary** and **sufficient** for the equilibrium of a rigid body, namely,  $\sum \vec{F} = 0$  and  $\sum \vec{M}_O = 0$ . When the body is subjected to a system of forces, which all lie in the x-y plane, then the forces can be resolved into their x and y components. Consequently, The **conditions for equilibrium** in two dimensions are

$$\begin{aligned}\sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M_O &= 0\end{aligned}$$

Here  $\sum F_x$  and  $\sum F_y$  represent, respectively, the **algebraic sums of the x and y components of all the forces** acting on the body, and  $\sum M_O$  represents **the algebraic sum of the couple moments and moments** of all the force components about the z axis, which is perpendicular to the x-y plan and passes through the arbitrary point O.

لنفترض جسم ما تحت تأثير عدد معين من القوى، فبعد تحليل كل القوى إلى مركباتها الأفقية والعمودية، يمكن كتابة معادلة الاتزان الجسم كالتالي:

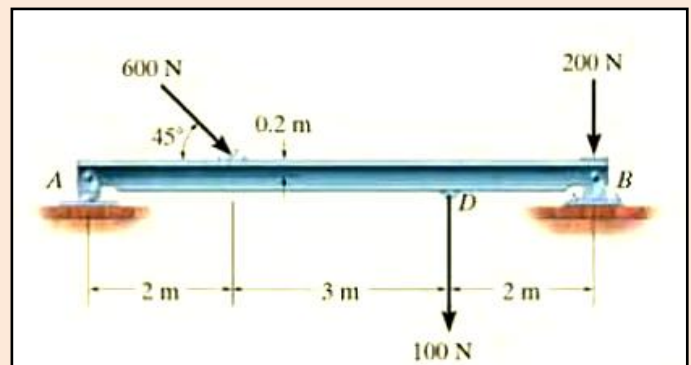
- مجموع القوى في اتجاه المحور x يساوي صفراً.
- مجموع القوى في اتجاه المحور y يساوي صفراً.
- عزم كل القوى (القوى الخارجية و ردود أفعال الدعامات) حول أي نقطة يساوي صفراً.

تعرف هذه المعادلات بمعادلات الاتزان أو قانون نيوتن للاتزان.

**Exercise 5.5:**

Determine the horizontal and vertical Components reaction on the beam caused by the pin at B and the rocker at A as shown in fig 5-7. Neglect the weight of the beam.

Fig 5-7



Ans:  $B_x = 424 \text{ N}$

$A_y = 319 \text{ N}$

$B_y = 405 \text{ N}$

**Exercise 5.6:**

The cord shown in fig 5-8 supports a force of 100 lb and wraps over the frictionless pulley. Determine the tension in the cord at C and the horizontal and vertical components of reaction at pin A.

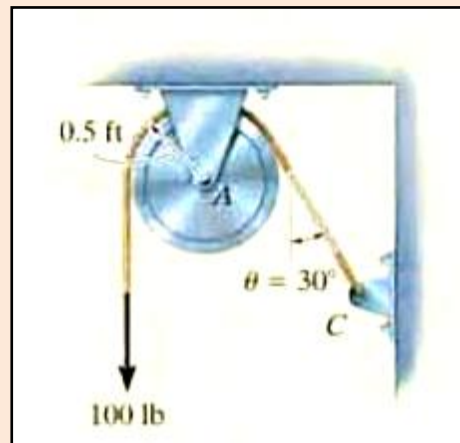


Fig 5-8

Ans:  $T = 100 \text{ lb}$        $A_x = 50 \text{ lb}$        $A_y = 187 \text{ lb}$

**Exercise 5.7:**

The member shown in fig 5-9 is pin-connected at A and rests against a smooth support at B. Determine the horizontal and vertical components of reaction at the pin A.

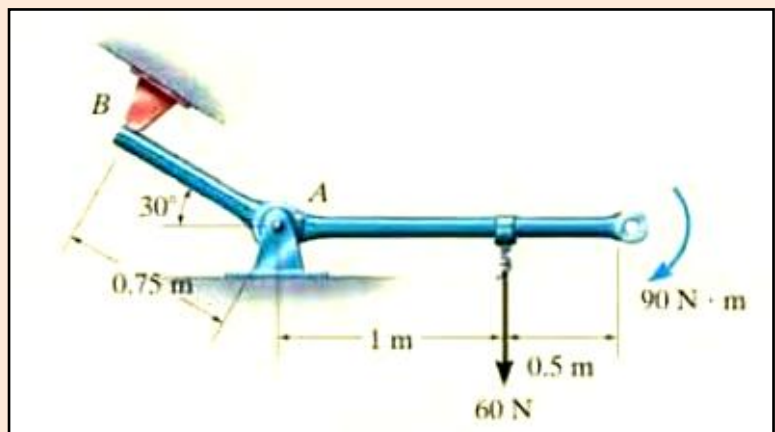


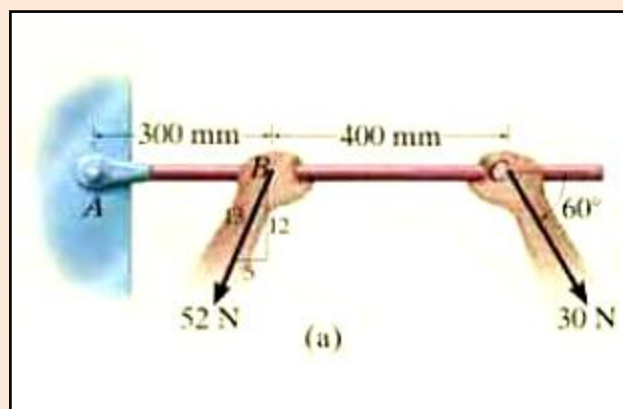
Fig 5-9

Ans:  $A_x = 100 \text{ N}$        $A_y = 233 \text{ N}$

**Exercise 5.8:**

The box wrench in fig. 5-10 is used to tighten the bolt at A. If the wrench does not turn when the load is applied to the handle, determine the torque or moment applied to bolt and the force of the wrench on the bolt.

Fig 5-10



Ans:  $A_x = 5 \text{ N}$        $A_y = 74 \text{ N}$        $M_A = 32.6 \text{ N.m}$        $F_A = 74.1 \text{ N}$

**Exercise 5.9:**

Determine the horizontal and vertical components of reaction on the member at the pin A, and the normal reaction at the roller B in fig 5-11.

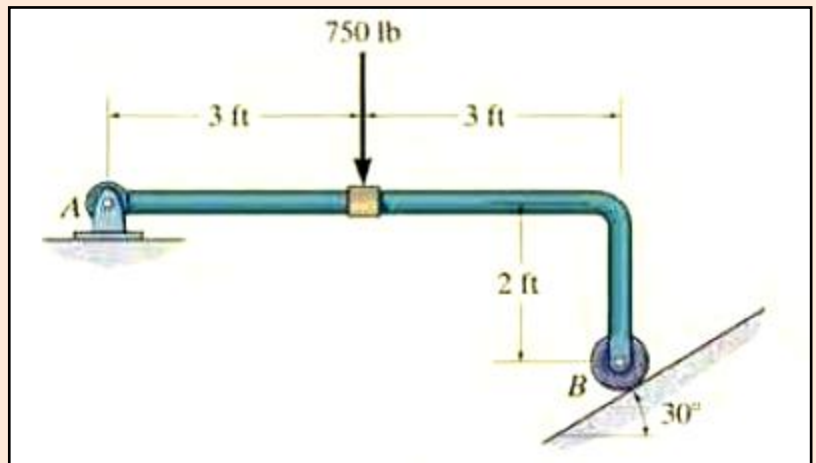


Fig 5-11

$$\text{Ans: } N_B = 536 \text{ lb} \quad A_x = 268 \text{ lb} \quad A_y = 286 \text{ lb}$$

**Exercise 5.10:**

The uniform smooth rod shown in fig 5-12 is subjected to a force and couple moment. If the rod is supported at A by a smooth wall and at B and C either at the top or bottom by rollers, determine the reactions at these supports. Neglect the weight of the rod.

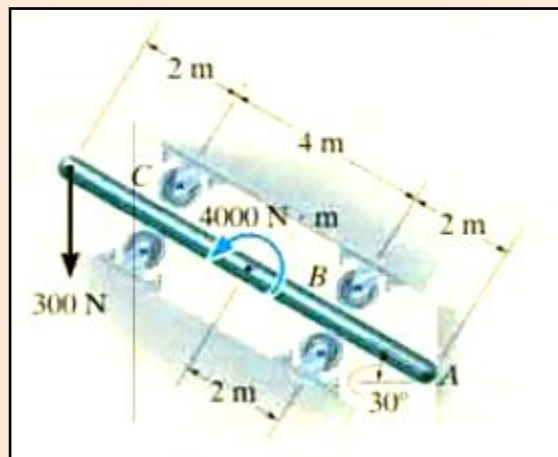


Fig 5-12

$$\text{Ans: } B_y = -1 \text{ kN} \quad C_y = 1.35 \text{ kN} \quad A_x = 173 \text{ N}$$

**Exercise 5.11:**

The uniform truck ramp shown in fig. 5-13 has a weight of 400 lb and is pinned to the body of the truck at each side and held in the position shown by the two side cables. Determine the tension in the cables.

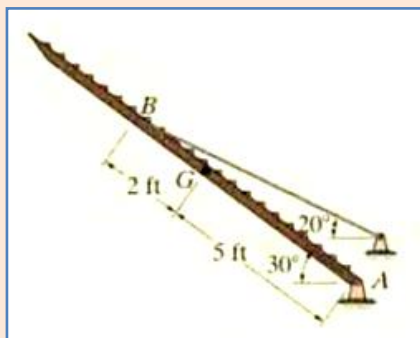


Fig 5-13

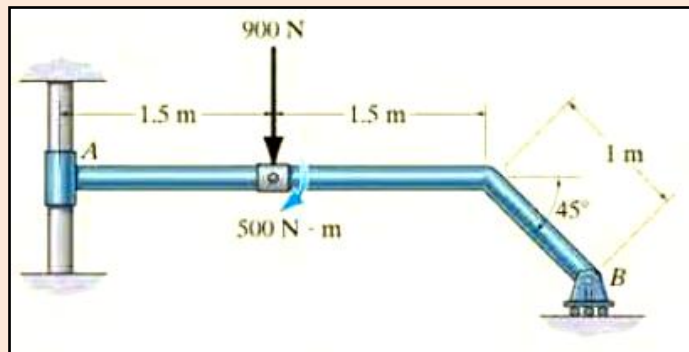


$$\text{Ans: } T' = \frac{T}{2} = 712 \text{ lb}$$

**Exercise 5.12:**

Determine the support reactions on the member in fig5-14. The collar at A is fixed to the member and can slide vertically along the vertical shaft.

Fig 5-14



Ans:  $A_x = 0 \text{ N}$

$N_B = 900 \text{ N}$

$M_a = 1.49 \text{ kN} \cdot \text{m}$

## Equilibrium in three dimensions

### 5.4 Free-body diagrams

It is first necessary to discuss the **types of reactions** that can occur at the **supports**.

#### Support reactions:








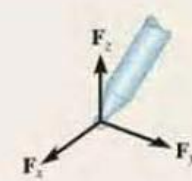
The reactive forces and couple moments acting at various **types of supports** and **connections** when the members are viewed in three dimensions are listed in **table 5-2** it is important to recognize the symbols used to represent each of these supports and to understand clearly how the forces and couple moments are developed. As in the two dimensional case:

- A force is developed by a support that restricts the translation of its attached member.
- A couple moments is developed when rotation of the attached member is prevented



الجدول التالي يعطي قائمة  
من الدعامات برؤية ثلاثية  
الأبعاد مع ردود الأفعال عند  
نقاط التماسها.

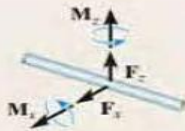
TABLE 5-2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
<p>(1)</p>  <p>cable</p>	 <p>One unknown. The reaction is a force which acts away from the member in the known direction of the cable.</p>	
<p>(2)</p>  <p>smooth surface support</p>	 <p>One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.</p>	
<p>(3)</p>  <p>roller</p>	 <p>One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.</p>	
<p>(4)</p>  <p>ball and socket</p>	 <p>Three unknowns. The reactions are three rectangular force components.</p>	

(5)



single journal bearing



Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. *Note:* The couple moments are generally not applied if the body is supported elsewhere. See the examples.

(6)



single journal bearing with square shaft

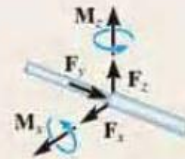


Five unknowns. The reactions are two force and three couple-moment components. *Note:* The couple moments are generally not applied if the body is supported elsewhere. See the examples.

(7)



single thrust bearing



Five unknowns. The reactions are three force and two couple-moment components. *Note:* The couple moments are generally not applied if the body is supported elsewhere. See the examples.

(8)



single smooth pin

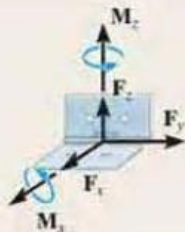


Five unknowns. The reactions are three force and two couple-moment components. *Note:* The couple moments are generally not applied if the body is supported elsewhere. See the examples.

(9)



single hinge

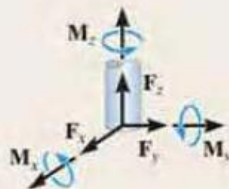


Five unknowns. The reactions are three force and two couple-moment components. *Note:* The couple moments are generally not applied if the body is supported elsewhere. See the examples.

(10)



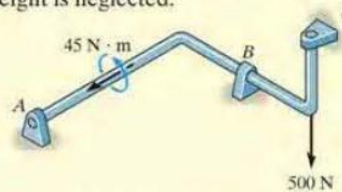
fixed support



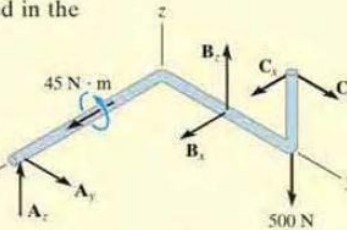
Six unknowns. The reactions are three force and three couple-moment components.



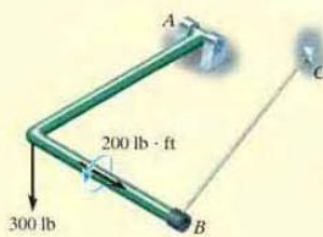
Consider the two rods and plate, along with their associated free-body diagrams shown in Fig. 5–23. The  $x$ ,  $y$ ,  $z$  axes are established on the diagram and the unknown reaction components are indicated in the *positive sense*. The weight is neglected.

**SOLUTION**

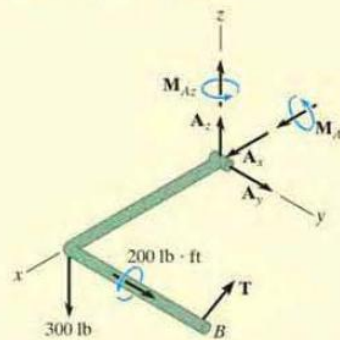
Properly aligned journal bearings at  $A$ ,  $B$ ,  $C$ .



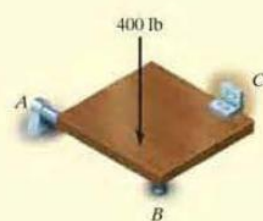
The force reactions developed by the bearings are sufficient for equilibrium since they prevent the shaft from rotating about each of the coordinate axes.



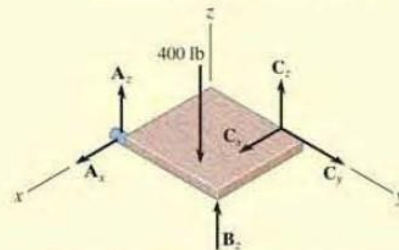
Pin at  $A$  and cable  $BC$ .



Moment components are developed by the pin on the rod to prevent rotation about the  $x$  and  $z$  axes.



Properly aligned journal bearing at  $A$  and hinge at  $C$ . Roller at  $B$ .



Only force reactions are developed by the bearing and hinge on the plate to prevent rotation about each coordinate axis. No moments at the hinge are developed.

## 5.5 Equations of Equilibrium

### Vector Equations of Equilibrium:

The two conditions for equilibrium of a rigid body may be expressed mathematically in vector form as

$$\sum \vec{F} = 0$$

$$\sum \vec{M}_O = 0$$

Where  $\sum \vec{F}$  is the vector sum of all the **external forces** acting on the body and  $\sum \vec{M}_O$  is the sum of the couple moments and the moments of **all the forces** about any point  $O$  located either on or off the body.

**Scalar Equations of Equilibrium:**

If all the **external forces and couple moments** are expressed in **Cartesian vector** form we have:

$$\sum \vec{F} = \sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_z \vec{k} = 0$$

$$\sum \vec{M}_o = \sum M_x \vec{i} + \sum M_y \vec{j} + \sum M_z \vec{k} = 0$$

Since the  $\vec{i}$ ,  $\vec{j}$  and  $\vec{k}$  components are independent from one another, the above Equations are satisfied provided.

$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \end{aligned}$$

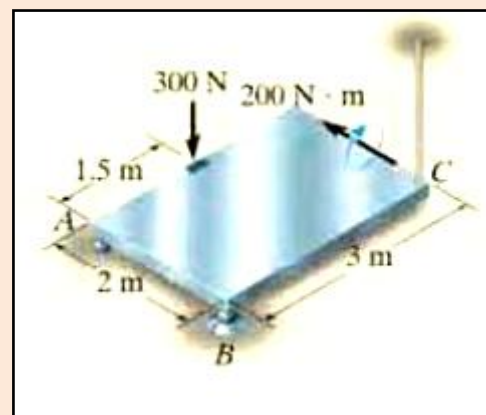
And

$$\begin{aligned} \sum M_x &= 0 \\ \sum M_y &= 0 \\ \sum M_z &= 0 \end{aligned}$$

بعد تحليل كل القوى إلى مركباتها السينية والعينية و  
الصادية، يمكن كتابة معادلة الاتزان الجسم كالتالي:

- مجموع القوى في اتجاه المحور x و المحور y و المحور z يساوي صفرا.
- مجموع عزوم القوى (القوى الخارجية و ردود أفعال الدعامات) في اتجاه المحاور الثلاث و حول أي نقطة يساوي صفرا.

Fig 5-15

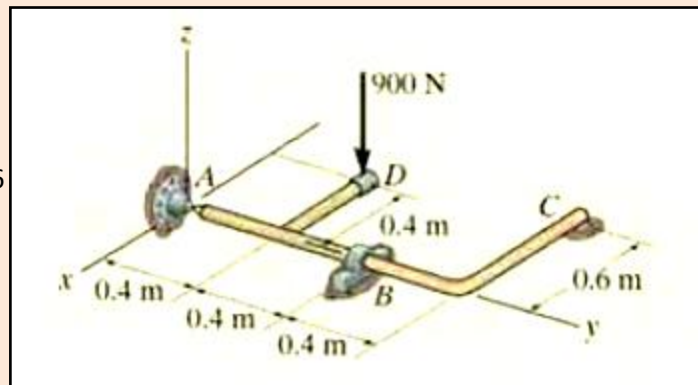


Ans:  $B_x = 0 \text{ N}$     $B_y = 0 \text{ N}$     $A_z = 790 \text{ N}$     $B_z = -217 \text{ N}$     $T_c = 707 \text{ N}$

**Exercise 5.14:**

Determine the components of reaction that the ball – and – socket joint at A, the smooth journal bearing at B, and the roller support at C exert on rod assembly in fig 5-16.

Fig 5-16

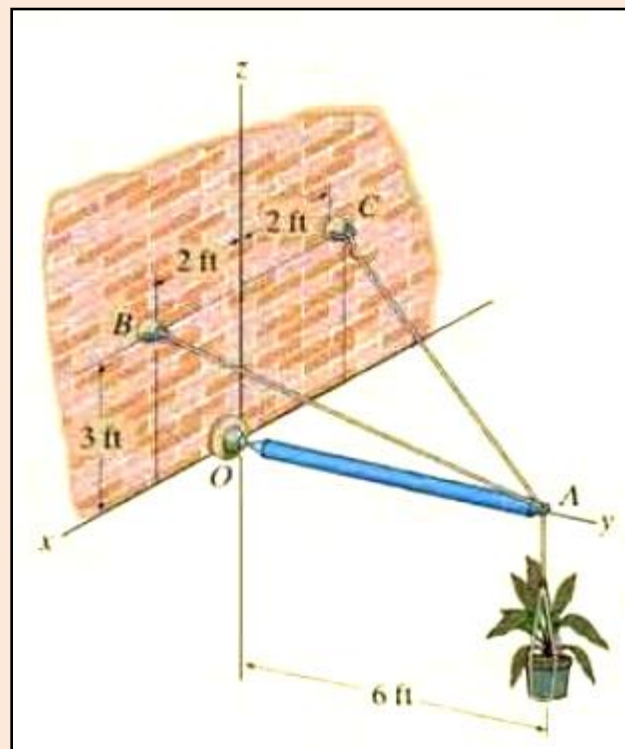


Ans:  $A_y = 0 \text{ N}$      $F_C = 600 \text{ N}$      $B_z = -450 \text{ N}$      $B_x = 0 \text{ N}$      $A_x = 0 \text{ N}$      $A_z = 750 \text{ N}$

**Exercise 5.15:**

The boom is used to support the 75 lb flowerpot in fig 5-17. Determine the tension developed in wires AB and AC.

Fig 5-17

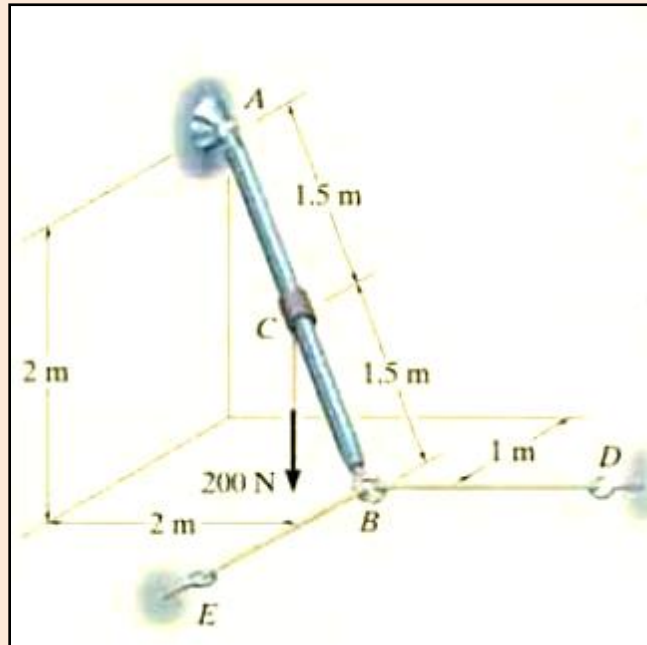


Ans:  $F_{AB} = F_{AC} = 87.5 \text{ lb}$

**Exercise 5.16:**

Rod AB shown in fig 5-18 is subjected to the 200 N force. Determine the reactions at the ball-and-socket joint A and the tension in the cables BD and BE

Fig 5-18



Ans:  $T_D = 100 \text{ N}$     $T_E = 50 \text{ N}$     $A_x = -50 \text{ N}$     $A_y = -100 \text{ N}$     $A_z = 200 \text{ N}$

**Exercise 5.17:**

The bent rod in Fig 5-19 is supported at A by a journal bearing, at D by a ball-and-socket joint, and at B by means of cable BC. Using only one Equilibrium Equation, obtain a direct solution for the tension in cable BC. The bearing at A is capable of exerting force components only in the z and y directions since it is properly aligned on the shaft.

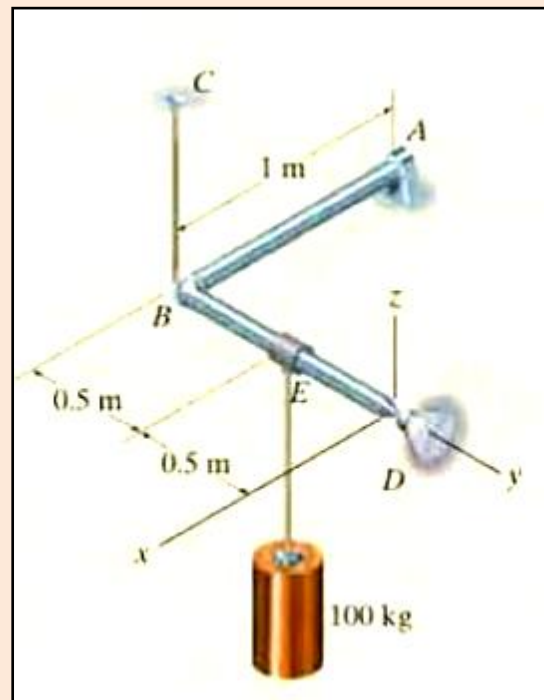


Fig 5-19

Ans:  $T_B = 490.5 \text{ N}$

# Chapter

Six

## Structural Analysis

فجمعك للعلم لا ينفع  
و علمك في الكتب مستودع

إذا لم تكن حافظا واعيا  
أتحضر بالجهل في مجلس

## Structural Analysis

### 6.1 Simple Trusses

A truss is a **structure composed of slender members joined together at their end points**. The members commonly used in construction consist of **wooden struts** or **metal bars**. In particular, **planar trusses** lie in a single plane and are often used to support **roofs** and **bridges**. The truss shown in Fig. 6-1a is an example of a typical roof-supporting truss. In this figure, the **roof load is transmitted to the truss at the joints**. Since this loading acts in the same plane as the truss, Fig. 6-1b, the analysis of the forces developed in the truss members will be two-dimensional.

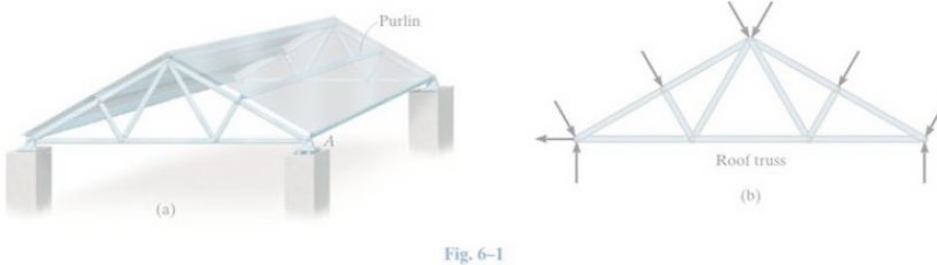


Fig. 6-1

In the case of a bridge, such as shown in Fig. 6-2a, the load on the deck is first **transmitted to stringers**, then to **floor beams**, and finally to the **joints** of the two supporting side trusses. Like the roof truss the bridge truss loading is also **coplanar**, Fig. 6-2b.

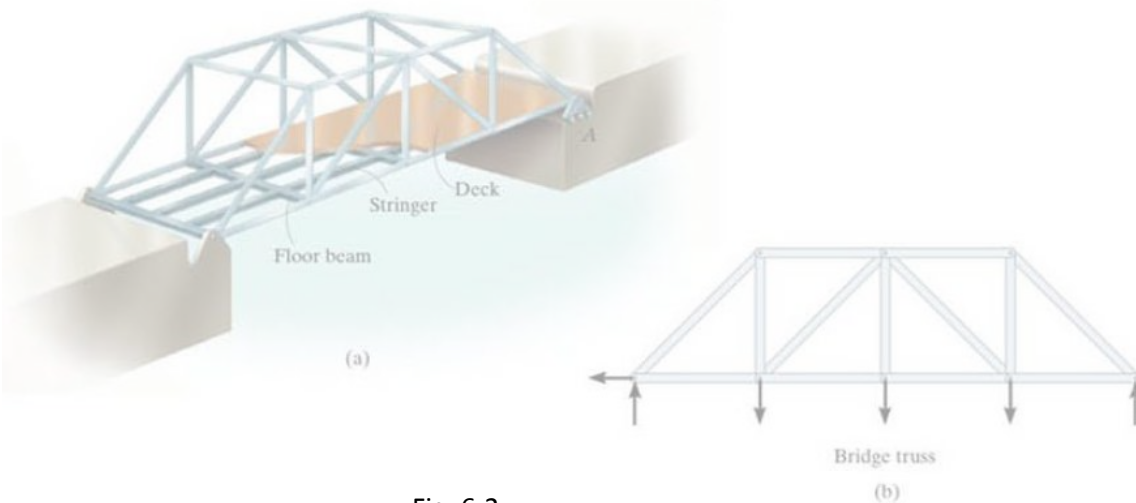


Fig. 6-2

الجمالون عبارة عن هيكل مكون من مجموعة من العناصر الطولية (القضبان) متصلة مع بعضها البعض عن طريق التلحيم أو المسامير الملولة أو التوصيلات المبرشمة بحيث تكون منشأ صلب مثل الجسر أو السقف... يكون وزن القضبان خفيفا بالنسبة للأحمال الواقعة عليها. نقطة التقاء العناصر الطولية تعرف باسم العقدة joint (أو المفاصل) و تكون الحمولة الخارجية المؤثرة على الجملون مركزة في العقد. العناصر الطولية المكونة للجملون تقاوم الحمولة عن طريق الشد أو الضغط و لا تقاوم العزم. إذا كانت القوة الداخلية للعنصر موجبة فإن العنصر في حالة شد وإذا كانت القوة الداخلية سالبة فإن العنصر في حالة ضغط.



**Assumptions for Design:**

To design both the members and the connections of a truss, it is necessary first to determine the force developed in each member when the truss is subjected to given loading. **To do this we will make two important assumptions:**

- **All Loadings are applied at the joints.**
- **the members are joined together by smooth pins.** The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a gusset plate as shown in Fig 6-3a. or by simply passing a large bolt or pin through each of the members. Fig 6-3b.



(a)



(b)

Fig. 6-3

تلتقي العناصر الطولية  
(القضبان) مع بعضها البعض  
عن طريق التلحيم أو المسامير  
المملولة أو التوصيلات المبرشمة  
في لوح الربط (gusset plate)

العناصر الطولية (القضبان)  
معرضة للشد (Tension) أو  
للضغط المحوري  
(compression) فهي تعمل  
إما كشدات أو سواند.

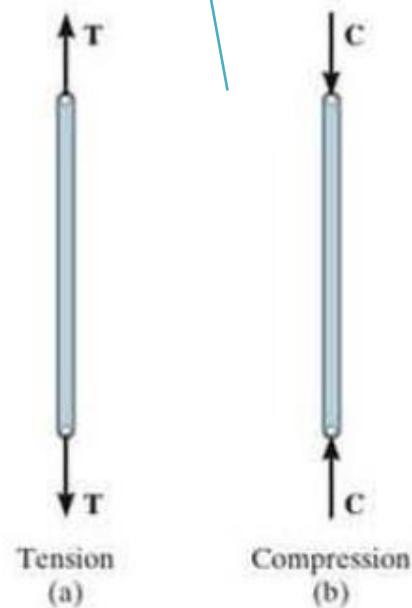
Tension  
(a)Compression  
(b)

Fig. 6-4

If the **force tends to elongate** the member, it is a **tensile force (T)**, Fig. 6-4a, whereas if it tends **to shorten** the member, it is a **compressive force (C)**, Fig. 6-4b.

If three members are pin connected at their ends they form a **triangular truss** that will be **rigid**, Fig. 6-5. Attaching two more members and connecting these members to a new joint D forms a larger truss. Fig. 6-6. This procedure can be repeated as many times as desired to form an even larger truss. If a truss can be constructed by expanding the basic triangular truss in this way, it is called a **simple truss**.

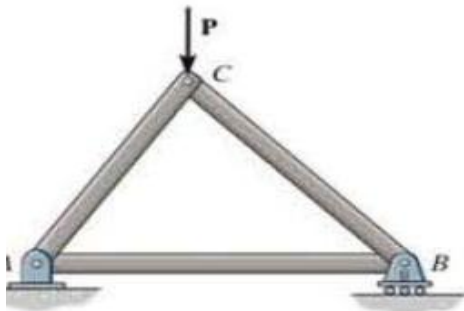


Fig. 6-5

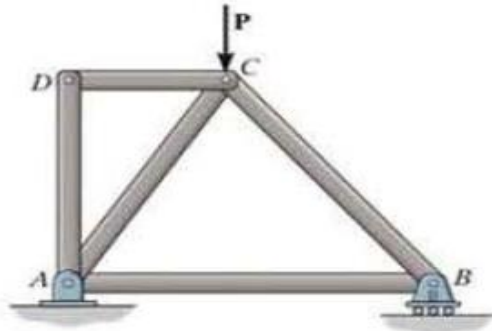


Fig. 6-6

## 6.2 The Method of Joints

In order to analyze or design a truss, it is necessary to determine the force in each of its members. One way to do this is to use the method of joints. This method is based on the fact that **if the entire truss is in equilibrium**, then each of its **joints is also in equilibrium**. Therefore, if the free-body diagram of each joint is drawn, **the force equilibrium equations can then be used to obtain the member forces acting on each joint**. Since the members of a **plane truss** are straight two-force members lying in a single plane, each joint is subjected to a force system that is **coplanar and concurrent**. As a result, only  $\sum F_x = 0$  and  $\sum F_y = 0$  need to be satisfied for equilibrium.

في طريقة العقد نقوم بعزل العقدة مع توضيح القوى الخارجية و الداخلية المؤثرة في العقدة حيث أن عناصر الجملون تكون في حالة شد أو ضغط (القوى الداخلية والتي هي على امتداد المحور الطولي للعنصر). وبما أن الجملون في حالة اتزان فإن كل عقدة في الجملون هي كذلك في حالة اتزان بحيث يمكن تطبيق معادلات الاتزان على العقدة وإيجاد القوى الداخلية في عناصر الجملون المتصلة مع العقدة. في كل عقدة يوجد معادلتين اتزان (محصلة القوى الأفقية تساوي صفر ومحصلة القوى العمودية تساوي صفر). وينصح في طريقة العقد بالبدء بالعقد التي لها عدد من المجاهيل يساوي 2 أو أقل (أي عدد العناصر يساوي 2 أو أقل).



For example, consider the pin at joint **B** of the truss in Fig. 6-7a. Three forces act on the pin, namely, the **500 N force** and the forces exerted by members **BA** and **BC**. The free body diagram of the pin is shown in Fig. 6-7b. Here  $\vec{F}_{BA}$  is "pulling" on the pin, which means that member **BA** is in **tension**; whereas  $\vec{F}_{BC}$  is "pushing" on the pin, and consequently member **BC** is in **compression**. These effects are clearly demonstrated by isolating the joint with small segments of the member connected to the pin, Fig. 6-7c. The pushing or pulling on these small segments indicates the effect of the member being either in compression or tension.

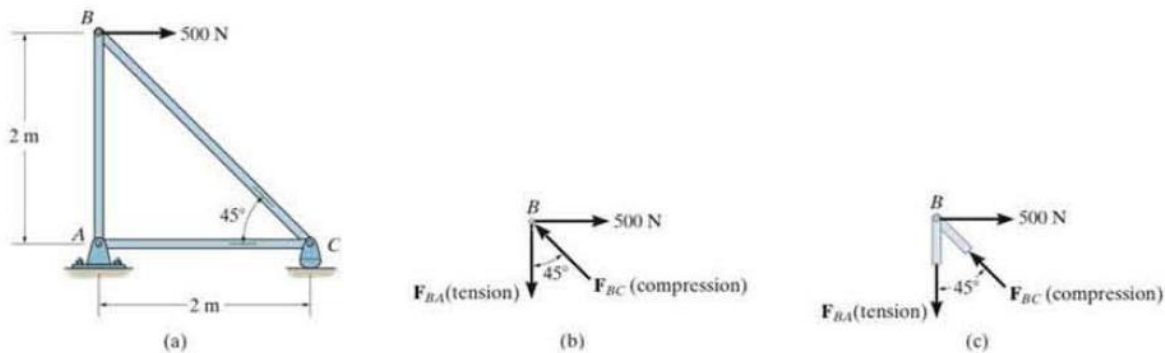
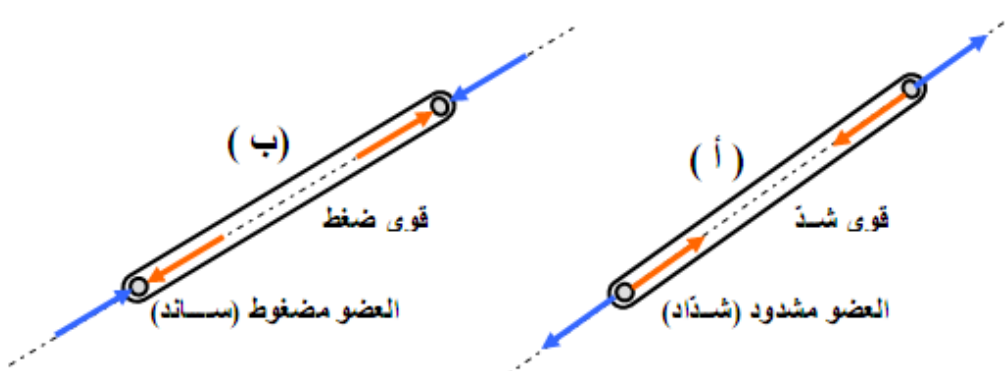


Fig. 6-7

Always assume the unknown member forces acting on the joint's Free-body diagram to be in **tension**: i.e., the forces "pull" on the pin. If this is done, then numerical solution of the equilibrium equations will yield **positive scalars** for members in **tension** and **negative scalars** for members in **compression**. Once an unknown member force is found, use its correct magnitude and sense (T or C) on subsequent joint free body diagrams.

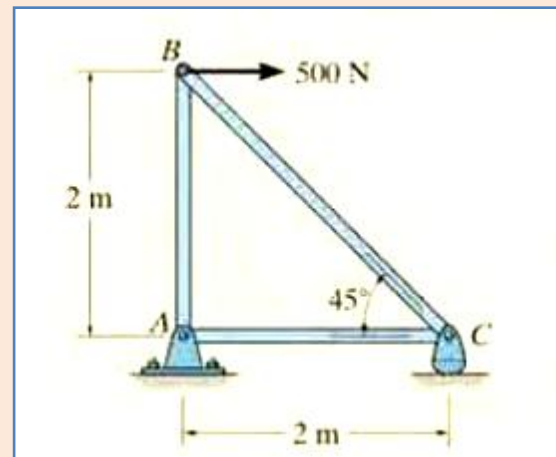
اتفاق الإشارات: نفترض بأن القوى المحورية الموجبة هي تلك التي تكون فيها الوصلة في حالة شد و سالبة في حالة الضغط.



**Exercise 6.1:**

Determine the force in each member of the truss shown in Fig. 6-8. and indicate whether the members are in tension or compression.

Fig. 6-8

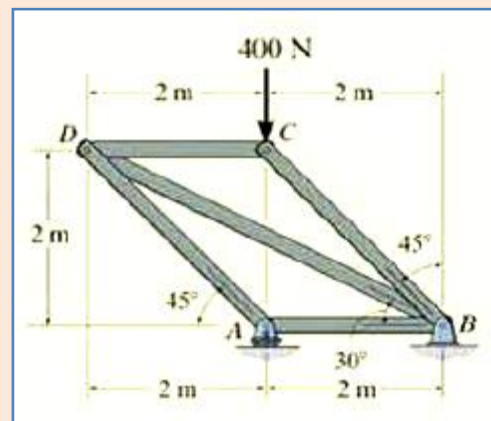


Ans:  $F_{BC} = 707.1 \text{ N}$      $F_{CA} = 500 \text{ N}$      $F_{BA} = 500 \text{ N}$      $C_y = 500 \text{ N}$

**Exercise 6.2:**

Determine the force in each member of the truss in Fig. 6-9 and indicate if the members are in tension or compression.

Fig. 6-9

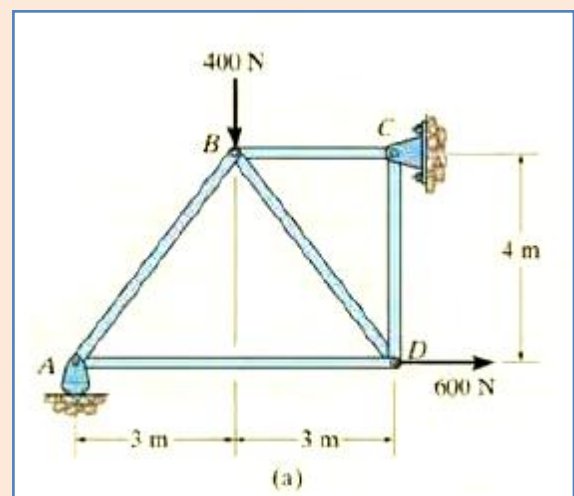


Ans:  $F_{BC} = 566 \text{ N (C)}$      $F_{CD} = 400 \text{ N (C)}$      $F_{AD} = 773 \text{ N (C)}$      $F_{BD} = 1.09 \text{ kN (T)}$      $F_{AB} = 546 \text{ N (C)}$

**Exercise 6.3:**

Determine the force in each member of the truss shown in Fig. 6-10. Indicate whether the members are in tension or compression.

Fig. 6-10



Ans:  $F_{AB} = 750 \text{ N (C)}$      $F_{AD} = 450 \text{ N (T)}$      $F_{BD} = 250 \text{ N (T)}$      $F_{DC} = 200 \text{ N (C)}$      $F_{CB} = 600 \text{ N (C)}$

## 6.3 Zero-Force Members

Truss analysis using the method of joints is **greatly simplified** if we can **first identify those members which support no loading**. These zero-force members are used to **increase the stability** of the truss during construction and to provide added support if the loading is changed.

If only **two members** form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero-force members.

القضبان عديمة القوى هي تلك  
القضبان التي تكون فيها القوى  
الداخلية المحورية تساوي صفر  
لذلك يمكن حذفها عند تحليل  
الجمالونات، ولا يمكن حذفها من  
الجمالون لكونها تحافظ على  
الاستقرار.

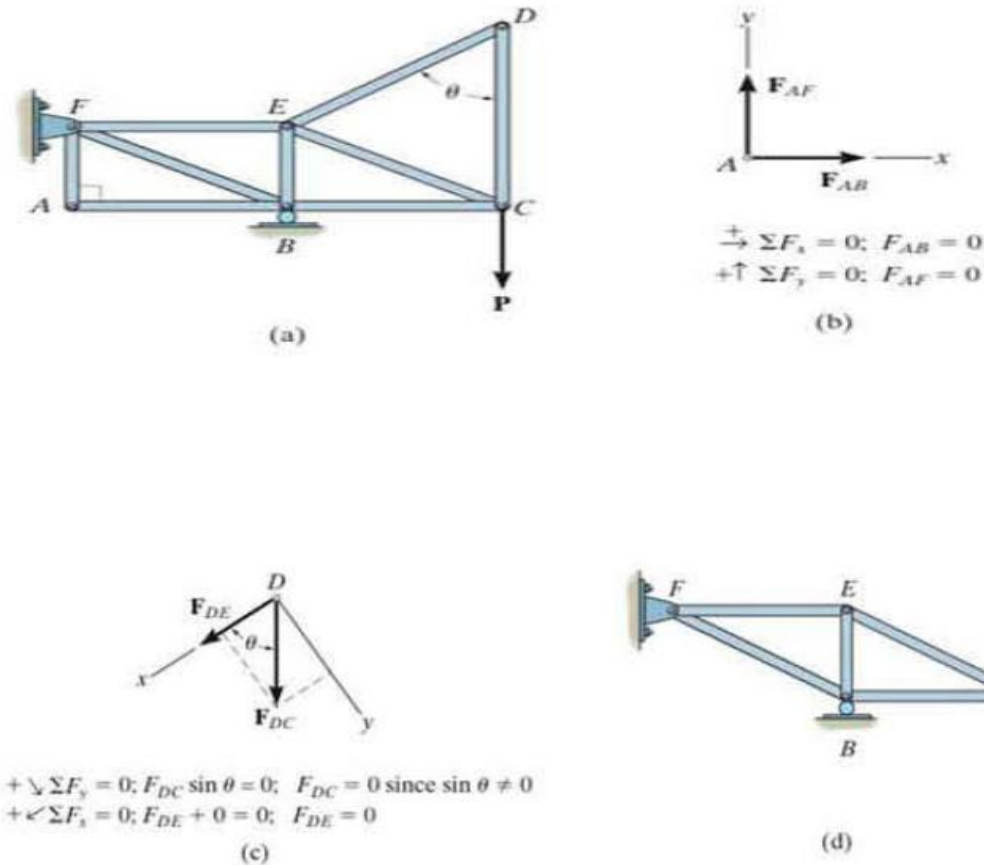
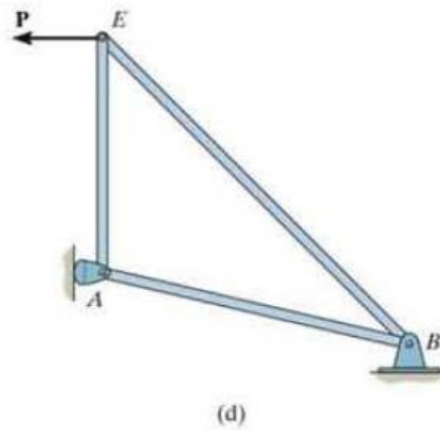
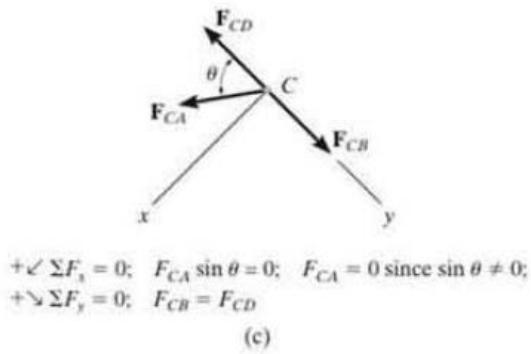
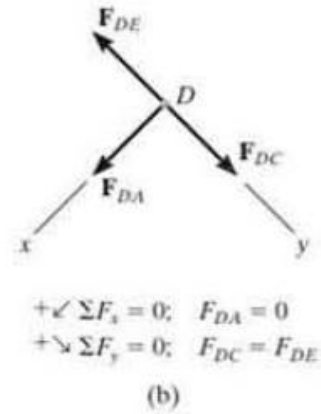
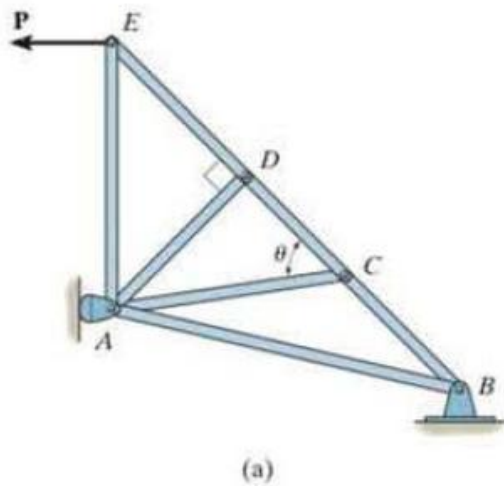


Fig. 6-11

if **three members** form a truss joint for which two of the members are **collinear**, the third member is a zero-force member provided no external force or support reaction is applied to the joint.

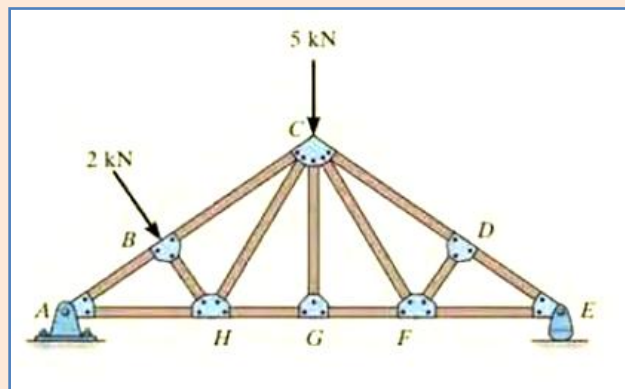


**Fig. 6-12**

### **Exercise 6.4:**

Using the method of joints, determine all the zero-force members of the Fink roof truss shown in Fig. 6-13. Assume all joints are pin connected.

Fig. 6-13



Ans:  $F_{GC} = 0 \text{ N}$

$$F_{DF} = 0 \text{ N}$$

$$F_{FC} = 0 \text{ N}$$

## 6.4 The Method of Sections

When we need to find the force in only a **few members** of a Truss. We can analyze the Truss, using the **method of sections**. It is based on the principle that if **the truss is in equilibrium** then any segment of the truss is also in equilibrium. For example, consider The two truss members shown on the Fig.6-14. If the forces within The members are to be determined, then **an imaginary section**, indicated by the blue line, can be used to cut each member into two parts and thereby "expose" each internal force as "external" to The free-body diagrams shown on The right. Clearly, it can be seen that equilibrium requires that the member in tension (T) be subjected to a "pull", whereas the member in compression (C) is subjected to a "push".

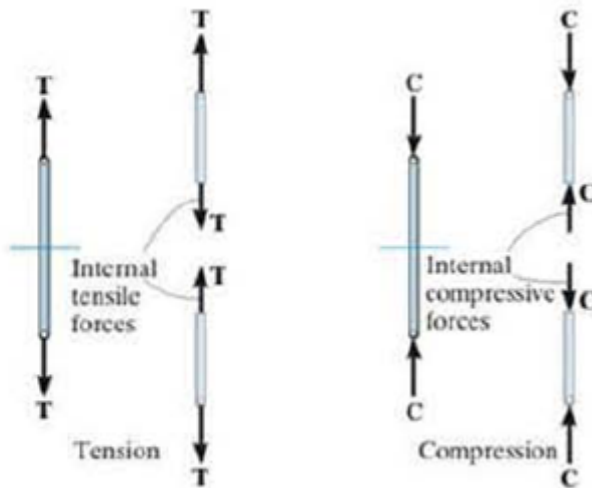


Fig. 6-14

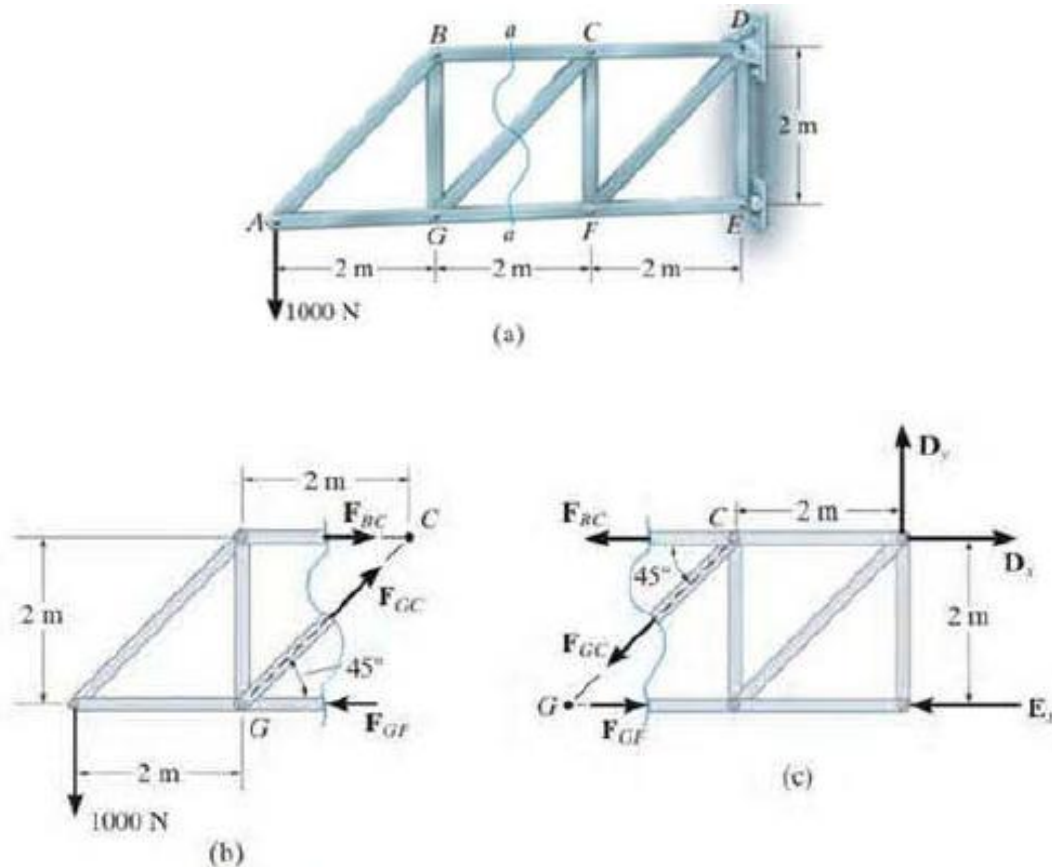
The method of sections can also be used to "cut" or section The members of an entire truss. If the section passes through the truss and the free-body diagram of either of its two parts is drawn, we can then apply the **equations of equilibrium** to that part to determine The member forces at the "**cut section**". Since only three independent equilibrium equations ( $\sum F_x = 0$ ,  $\sum F_y = 0$ ,  $\sum M_0 = 0$ ) can be applied to the free-body diagram of any segment, then we should try to **select a section** that, in general, passes through not more than three members in which the forces are unknown.

For example, consider the truss in Fig. 6-15a. If the forces in members BC, GC, and GF are to be determined, then section aa would be appropriate. The free -body diagrams of the two segments are shown in Figs 6-15b and 6-15c. Note that the line of action of each member force is specified from the geometry of the truss, since the force in a member is along its axis. Also, the member forces acting on one part of the truss are equal but opposite to those acting on the other part - Newton's third law. Members BC and GC are assumed to be in tension since they are

ترتكز طريقة المقاطع على مبدأ الجسم المقتطع، تتلخص في أخذ مقطع وهمي يقسم الجملون إلى جزأين متزئين و منفصلين. يجب أن يراعى في اختيار المقطع أن لا يقطع أكثر من ثلاثة قضبان مجهولة القوى الداخلية. يمكن عندئذ حساب القوى الداخلية في القضبان المقطوعة بكتابة معادلات الاتزان الثلاثة لأي من الجزأين وذلك بعد حساب ردود الأفعال على الجملون ككل.

subjected to a "pull", whereas GF in compression since it is subjected to a "push".

The three unknown member forces  $F_{BC}$ ,  $F_{GC}$  and  $F_{GF}$  can be obtained by applying the three equilibrium equations to the free-body diagram in Fig. 6-15b. If however, the free-body diagram in Fig. 6-15c is considered, the three support reactions  $D_x$ ,  $D_y$  and  $E_x$  will have to be known, because only three equations of equilibrium are available. (This, of course, is done in the usual manner by considering a free-body diagram of the entire truss.)



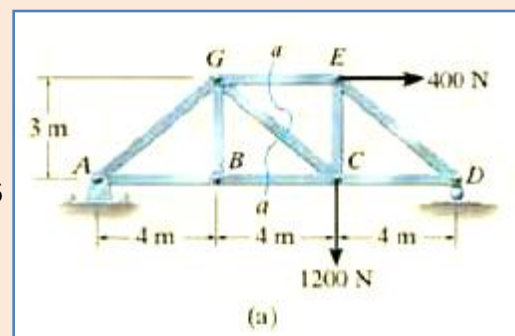
**Fig. 6-15**

**Exercise 6.5:**

Determine the force in members GE, GC, and BC of the truss shown in Fig. 6-16.

Indicate whether the members are in tension or compression.

**Fig. 6-16**



Ans:  $F_{BC} = 800 \text{ N (T)}$

$F_{GE} = 800 \text{ N (C)}$

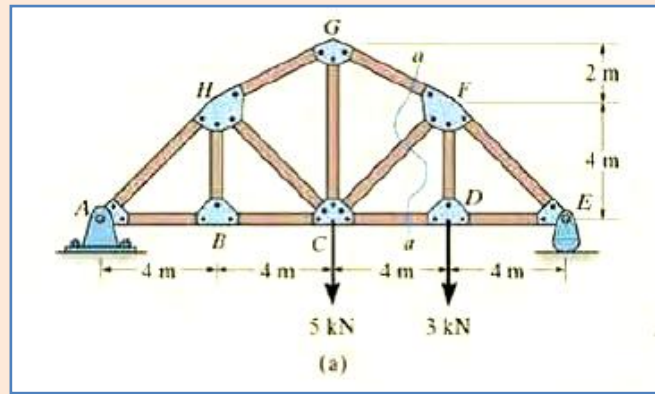
$F_{GC} = 500 \text{ N (T)}$



**Exercise 6.6:**

Determine the force in member CF of the truss shown in Fig. 6-17. Indicate whether the member is in tension or compression. Assume each member is pin connected.

Fig. 6-17

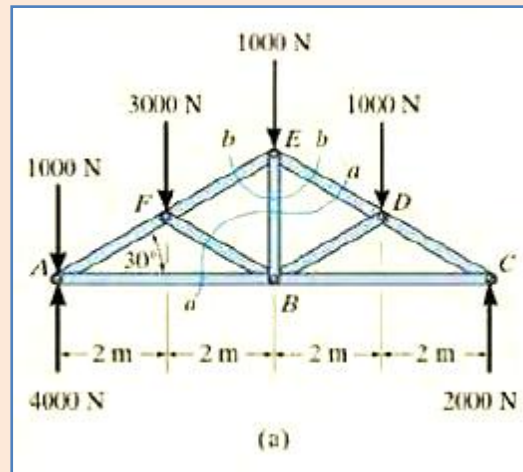


$$\text{Ans: } F_{CF} = 0.589 \text{ kN (C)}$$

**Exercise 6.7:**

Determine the force in member EB of the roof truss shown in Fig. 6-18. Indicate whether the member is in tension or compression.

Fig. 6-18



$$\text{Ans: } F_{CF} = 0.589 \text{ kN (C)}$$

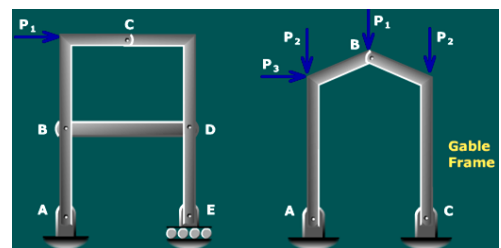
**6.5 Frames and Machines**

هياكل و ماكينات

**Frames and machines** are structures that contain **pin-connected multi-force members** (members with more forces or two forces couple or couples acting on it).

**Frames** (sign frame, building frame, etc.) are **used** to **support the system of loads** while remaining stationary.

**Machines** (pliers, front end loaders, back hoists, etc.) **contain moving parts** and are **designed to transmit** or **modify loads**.



In the **analysis** of statically **determinate frames** generally begins with consideration of **equilibrium** of the overall frame to **determine support reactions**. In order to **determine the forces** acting at the **joints** and **supports** a frame or machine, the **structure** must be **disassembled** and the **free body diagrams** of its parts must be drawn.

**Exercise 6.8:**

For the frame shown in Fig. 6-19, draw the free-body diagram of (a) each member, (b) the pin at B, and (c) the two members connected together.

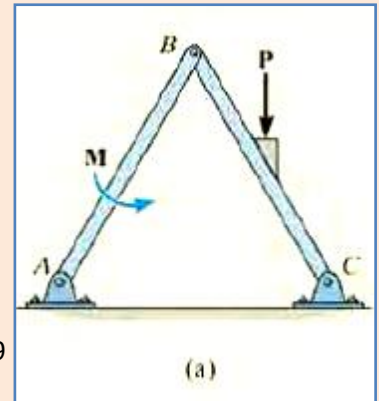
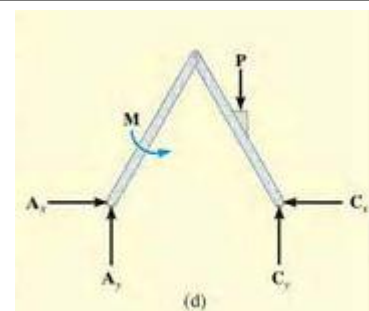
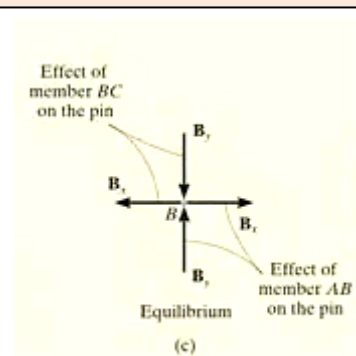
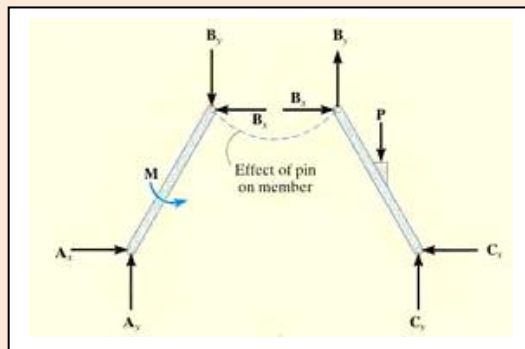


Fig. 6-19

Ans:





**Exercise 6.9:**

A constant tension in the conveyor belt is maintained by using the device shown in Fig. 6-20. Draw the free-body diagrams of the frame and the cylinder that the belt surrounds. The suspended block has a weight of  $W$ .

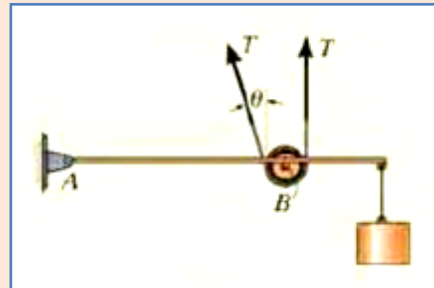
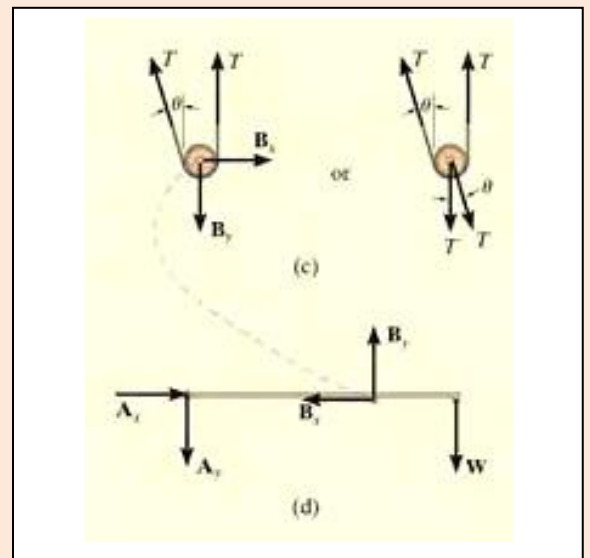


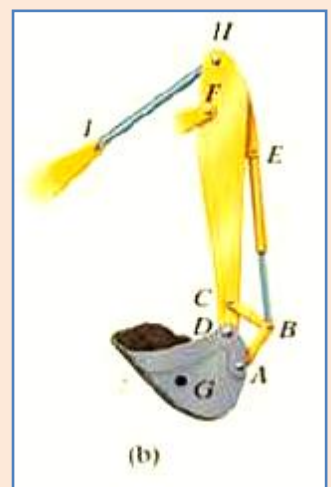
Fig. 6-20

Ans:

**Exercise 6.10:**

Draw the free-body diagrams of the bucket and the vertical boom of the backhoe shown in the photo, Fig. 6-21. The bucket and its contents have a weight  $W$ . Neglect the weight of the members.

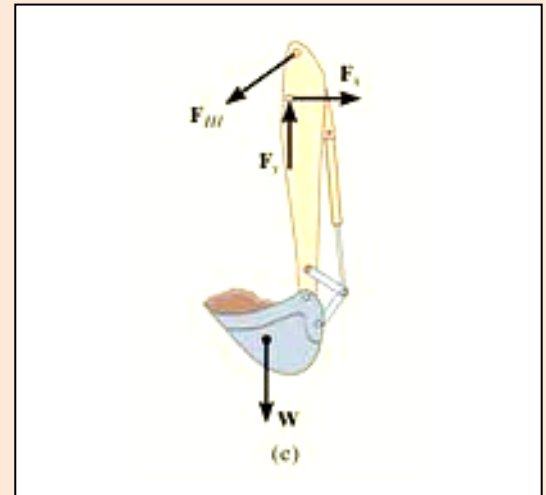
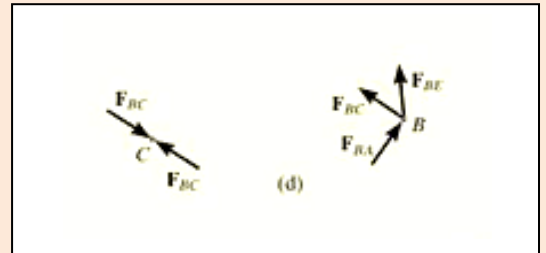
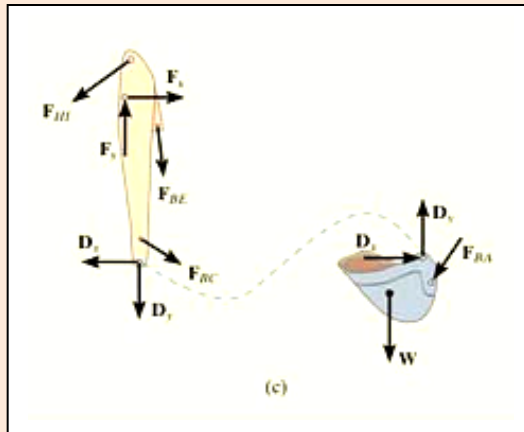
Fig. 6-21



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Ans:



# Chapter

Seven

## Internal Forces

اعلم أخي الطالب أن:

العلم يشرح الصدر و يوسع مدارك النظر و يفتح الأفاق أمام  
النفس فتخرج من همها و غمها و حزنها

## Internal Forces

### 7.1 Internal Forces Developed in Structural Members:

To design a structural or mechanical member it is necessary to know the loading acting within the member in order to be sure the material can resist this loading. Internal loadings can be determined by using the **method of sections**. To illustrate this method, consider the cantilever beam in Fig. 7- 1a. If the internal loadings acting on the cross section at point B are to be determined, we must pass an imaginary section a-a perpendicular to the axis of the beam through point B and then separate the beam into two segments. The internal loadings acting at B will then be exposed and become external on the free-body diagram of each segment, fig. 7- 1b.

الغرض الرئيسي من دراسة القوى الداخلية للعناصر الإنشائية هو التحقق من قدرته على تحمل كافة القوى التي يتعرض لها دون أن يتصدع أو ينهار.

يتم دراسة القوى الداخلية بتطبيق طريقة المقاطع التي تفصل العنصر الإنشائي إلى جزأين متزيين يخضعن إلى:

- قوة مماسة للمقطع  $V_B$ .
- قوة محورية عمودية على المقطع  $(N_B)$ .
- عزم إنحاء  $(M_B)$ .

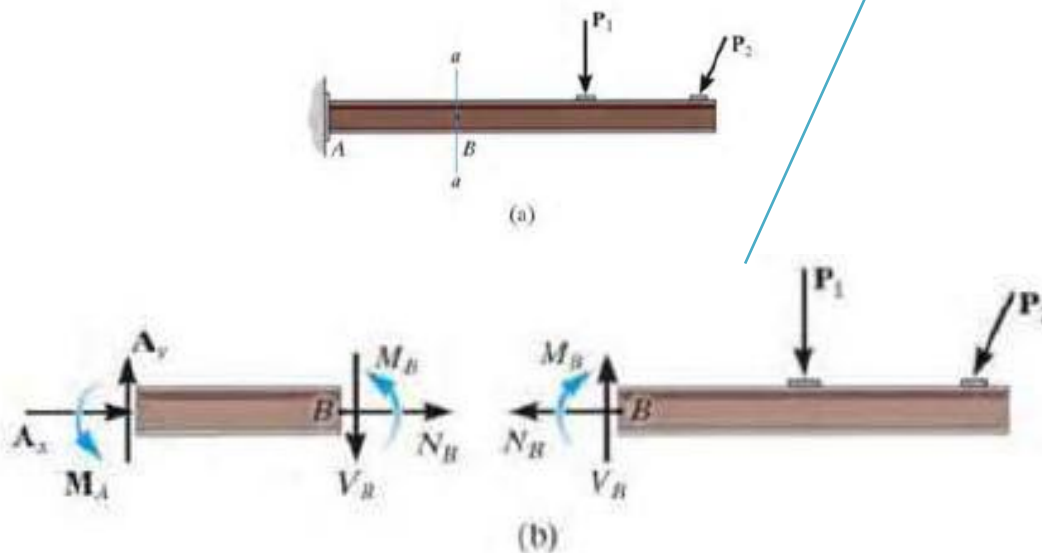


Fig. 7-1

The force component  $\vec{N}_B$  that acts **perpendicular** to the cross section is termed the **normal force**. The force component  $\vec{V}_B$  that is tangent to the cross section is called the **shear force** and the couple moment  $\vec{M}_B$  is referred to as the **bending moment**. The force components prevent the relative translation between the two segments. and the couple moment prevents the relative rotation. According to Newton's third law, these loadings must act in opposite directions on each segment. as shown in Fig. 7-1b. They can be determined by applying the equations of equilibrium to the free-body diagram of either segment. In this case, however, the right segment is the better choice since it does not involve the unknown support reactions at A. A direct solution for  $\vec{N}_B$  is obtained by applying  $\sum F_x = 0$ ,  $\vec{V}_B$  is obtained from  $\sum F_y = 0$ . and  $\vec{M}_B$  can be obtained by applying  $\sum M_B = 0$ , since the moments of  $\vec{N}_B$  and  $\vec{V}_B$  about B are zero.

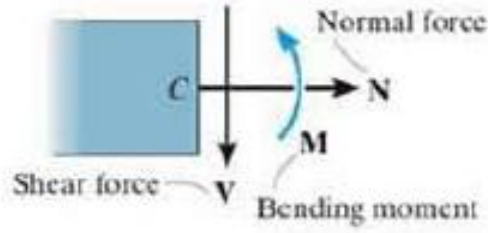
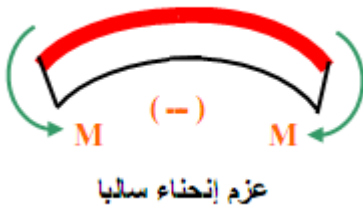
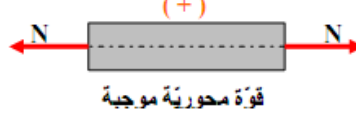
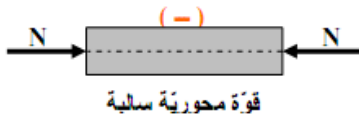
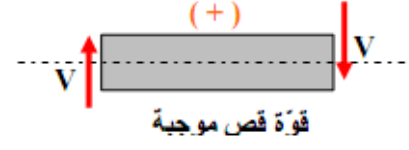
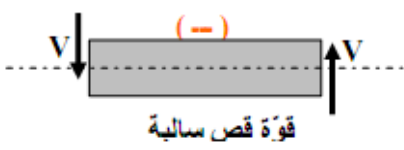


Fig 7-2



القوة المماسية للمقطع ( $V_B$ ) وتسمى قوة القص عند مقطع محدد وهي القوة التي تؤثر عموديا على المحور الطولي للعنصر الإنشائي، وتمثل مجموع المركبات الرأسية لجميع القوى الخارجية، بما فيها ردود الأفعال. وتعتبر قوة القص موجبة إذا كانت إلى أعلى عن يسار المقطع أو إلى أسفل عن يمين المقطع، وسالبة في حالة عكس ذلك.

القوة المحورية العمودية على المقطع ( $N_B$ ) هي القوة التي تؤثر في الاتجاه المحور الطولي للعنصر الإنشائي وتمثل مجموع المركبات الأفقية لجميع القوى الخارجية بما فيها ردود الأفعال. وتعتبر موجبة إذا كانت شادة للمقطع وسالبة إذا كانت ضاغطة له.

عزم الانحناء ( $M_B$ ) عند مقطع من المقاطع هو مجموع عزوم جميع القوى الخارجية، بما فيها ردود الأفعال التي تقع إما عن يمين المقطع فقط أو يساره فقط، وذلك حول مركز المقطع. وتعتبر عزم الانحناء الذي يتسبب فيه شد في الألياف السفلية أو ضغطا في الألياف العلوية للعنصر الإنشائي موجبا، وسالبا في حالة عكس ذلك.

### Sign Convention:

Engineers generally use a sign convention to report the three internal loadings  $\vec{N}$ ,  $\vec{V}$ , and  $\vec{M}$ . Although this sign convention can be arbitrarily assigned, the one that is widely accepted will be used here, Fig. 7-3. The normal force is said to be positive if it creates **tension**, a positive shear force will cause the beam segment on which it acts to rotate clockwise, and a positive bending moment will tend to bend the segment on which it acts in a concave upward manner. Loadings that are opposite to these are considered negative.



Positive normal force



Positive shear



Positive moment



Fig. 7-3

### Procedure for Analysis:

The method of sections can be used to determine the internal loadings on the cross section of a member using the following procedure.

### Support Reactions:

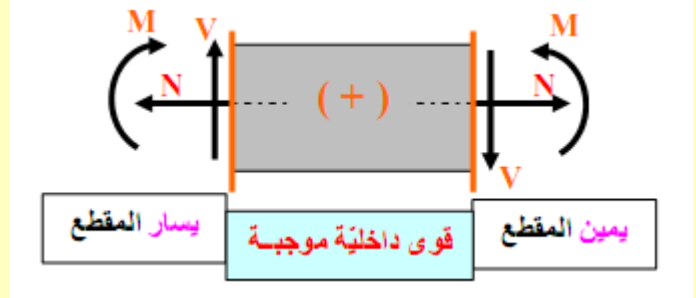
- Before the member is sectioned, it may first be necessary to determine its support reactions, so that the equilibrium equations can be used to solve for the internal loadings only after the member is sectioned.

### Free-body Diagram:

- Keep all distributed loadings, couple moments, and forces acting on the member in their exact locations, then pass an imaginary section through the member, perpendicular to its axis at the point where the internal loadings are to be determined.

- After the section is made, draw a free-body diagram of the segment that has the least number of loads on it, and indicate the components of

نلخص قواعد إشارات قوى القص والمحورية وعزم الانحناء، المتفق عليه كما يلي:



خطوات تحليل الكمرات البسيطة:

الخطوة الأولى:

إيجاد ردود أفعال وذلك باستعمال معادلات الاتزان بعد رسم مخطط جسم الحر.

الخطوة الثانية:

إيجاد قيمة القوى الداخلية (قوى القص و العزم) في النقاط الحرجة من الكمرة على امتداد المحور الأفقي  $x$ . ويمكن عمل هذه الخطوة بقص الكمرة في أي نقطة ثم رسم مخطط الجسم الحر لكل جزء من الكمرة. القوى الداخلية تنقسم إلى قوة القص وهي عمودية للكمرة وقوة شد داخلية وهي أفقية بالنسبة للكمرة. يتم حساب قوى العزم حول النقطة الحرجة بحساب مجموع العزوم حول نفس النقطة. محصلة العزوم حول النقطة الحرجة يجب أن تكون تساوي صفر.

the internal force and couple moment resultants at the cross section acting in their positive directions to the established sign convention.

### Equations of Equilibrium:

- Moments should be summed at the section. This way the normal and shear forces at the section are eliminated, and we can obtain a direct solution for the moment.
- If the solution of the equilibrium equations yields a negative scalar, the sense of the quantity is opposite to that shown on the free-body diagram.

#### **Exercise 7.1:**

Determine the normal force, shear force, and bending moment acting just to the left of point B, and just to the right, point C, of the 6-kN force on the beam in Fig. 7-4.

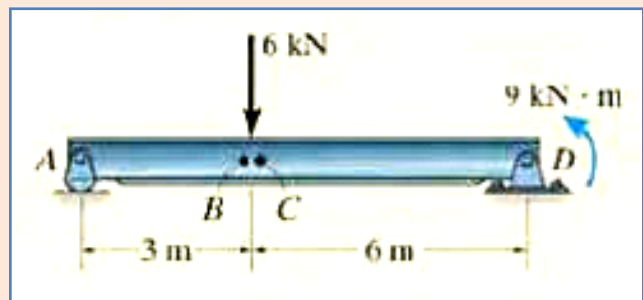


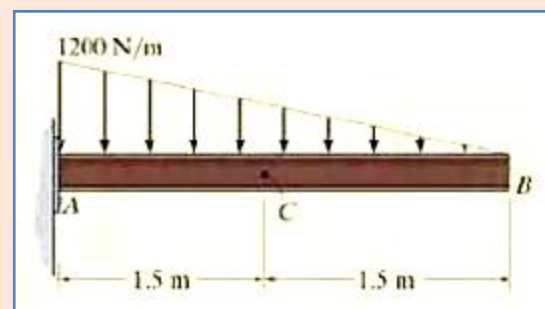
Fig. 7-4

Ans: $N_B = 0 \text{ N}$	$V_B = 5 \text{ kN}$	$M_B = 15 \text{ kN}\cdot\text{m}$
$N_C = 0 \text{ N}$	$V_C = -1 \text{ kN}$	$M_C = 15 \text{ kN}\cdot\text{m}$

#### **Exercise 7.2:**

Determine the normal force, shear force, and bending moment at C of the beam in Fig. 7-5.

Fig. 7-5



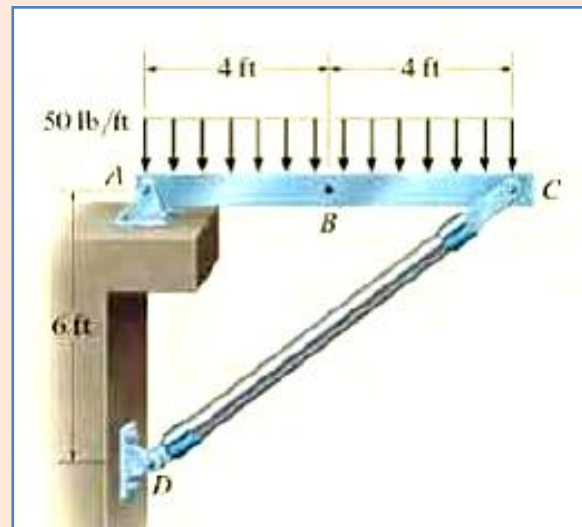
Ans: $N_C = 0 \text{ N}$	$V_C = 450 \text{ N}$	$M_C = -225 \text{ N}\cdot\text{m}$
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**Exercise 7.3:**

Determine the normal force, shear force, and bending moment acting at point B of the two-member frame shown in Fig. 7-6.

Fig. 7-6

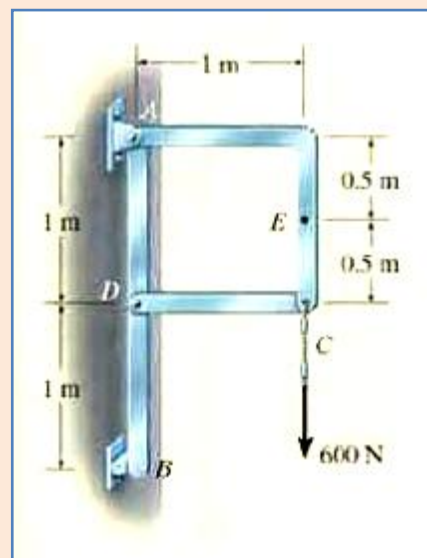


Ans:  $N_B = 267 \text{ lb}$        $V_B = 0 \text{ lb}$        $M_B = 400 \text{ lb.ft}$

**Exercise 7.4:**

Determine the normal force, shear force, and bending moment acting at point E of the frame loaded as shown in fig. 7-7.

Fig. 7-7



Ans:  $V_E = 600 \text{ N}$        $N_E = 600 \text{ N}$        $M_E = 300 \text{ N.m}$



**7.2 Shear and Moment Equations and Diagrams**

**beams** are structural members designed to support loadings applied perpendicular to their axes. In general, they are long and straight and have a constant cross-sectional area. They are often classified as to how they are supported. For example, a **simply supported beam** is pinned at one end and roller supported at the other, as in Fig. 7-8a, whereas a **cantilevered beam** is fixed at one end and free at the other. The actual design of a beam requires a detailed knowledge of the **variation** of the internal shear force  $V$  and bending moment  $M$  acting at **each point** along the axis of the beam.

these **variations** of  $V$  and  $M$  along the beam's axis can be obtained by using the method of sections discussed in Sec. 7.1. In this case, however, it is necessary to section the beam  $M$  at an arbitrary distance  $x$  from one end and then apply the equations of equilibrium to the segment having the length  $x$ . Doing this we can then obtain  $V$  and  $M$  as functions of  $x$ .

In general, the internal shear and bending-moment functions will be discontinuous, or their slopes will be discontinuous, at points where a distributed load changes or where concentrated forces or couple moments are applied. Because of this, these functions must be determined for each **segment** of the beam located between any two discontinuities of loading. For example, segments having lengths  $x_1$ ,  $x_2$ , and  $x_3$  will have to be used to describe the variation of  $V$  and  $M$  along the length of the beam in Fig. 7-8a. These functions will be valid only within regions from  $O$  to  $a$  for  $x_1$ , from  $a$  to  $b$  for  $x_2$ , and from  $b$  to  $L$  for  $x_3$ . If the resulting functions of  $x$  are plotted, the graphs termed the **shear diagram** and **bending-moment diagram**, Fig. 7-8b and Fig. 7-8c, respectively.

تخطيط القوى الداخلية:

لتمثيل القوى الداخلية نعتبر

محور العنصر الإنشائي هو خط

القاعدة. تمثل الإحداثيات

العمودية لقيم القوى الداخلية

في العنصر الإنشائي بالنسبة لهذا

الخط. ويتم حساب تغيراتها عند

عدد من المقاطع من خلال

تعبيرات جبرية أو دالات،

وبنفس الطرق التي تم شرحها

سابقاً.

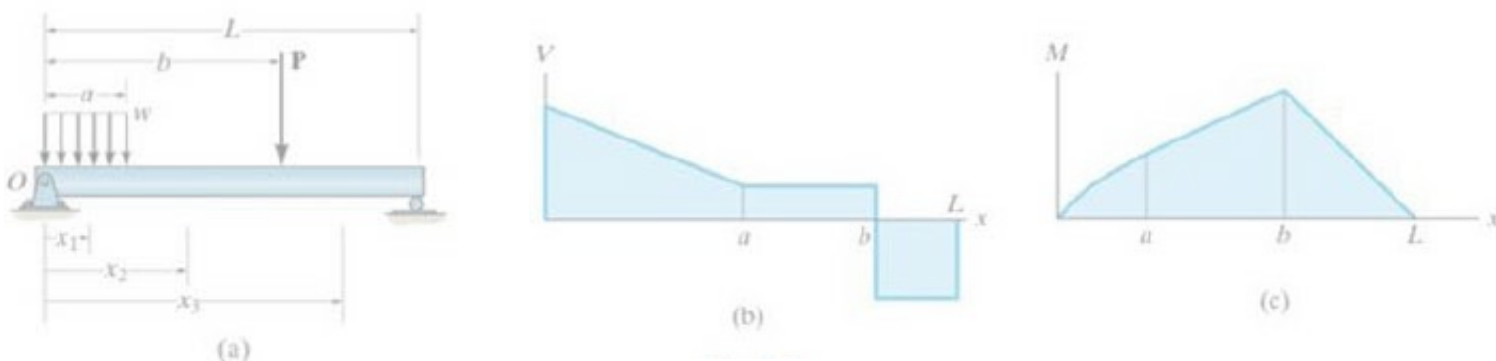


Fig. 7-8

**Procedure for Analysis**

The shear and bending-moment diagrams for a beam can be constructed using the following procedure.

يتم حساب أولا ردود الأفعال

**Support Reactions:**

- Determine all the reactive forces and couple moments acting on the beam and resolve all the forces into components acting perpendicular and parallel to the beam's axis.

**Shear and Moment Functions:**

- Specify separate coordinates  $x$  having an origin at the beam's left end and extending to regions of the beam between concentrated forces and/or couple moments, or where the distributed loading is continuous.
- Section the beam at each distance  $x$  and draw the free-body diagram of one of the segments. Be sure  $V$  and  $M$  are shown acting in their positive sense, in accordance with the sign convention given in fig. 7-9.
- The shear  $V$  is obtained by summing forces perpendicular to the beam's axis.
- the moment  $M$  is obtained by summing moments about the sectioned end of the segment.

تحلل الكمرة بعد ذلك باعتبار مقطع واحد متحرك على كامل العنصر الإنشائي من اليسار إلى اليمين موضعه  $x$ ، ثم تطبق شروط الاتزان لإيجاد معادلات القوى الداخلية والقص و عزم الانحناء بدلالة  $x$ . يتم رسم التخطيطات بعد ذلك بسهولة.

**Shear and Moment Diagrams:**

- Plot the shear diagrams ( $V$  versus  $x$ ) and the moment diagram ( $M$  versus  $x$ ). If computed values of the functions describing  $V$  and  $M$  are positive, the values are plotted above the  $x$  axis, whereas negative values are plotted below the  $x$  axis.
- Generally, it is convenient to plot the shear and bending-moment diagrams directly below the free-body diagram of the beam.

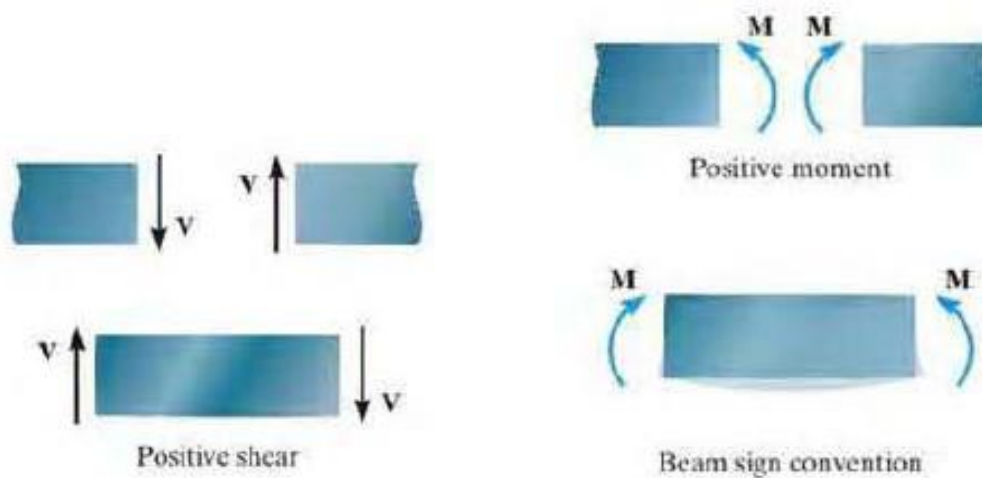
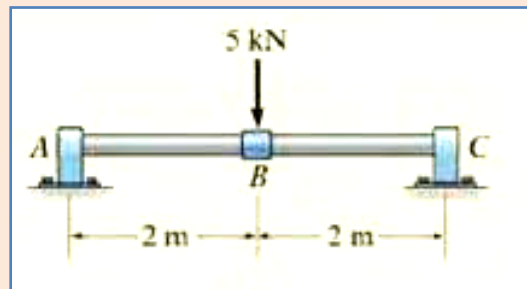


Fig. 7-9

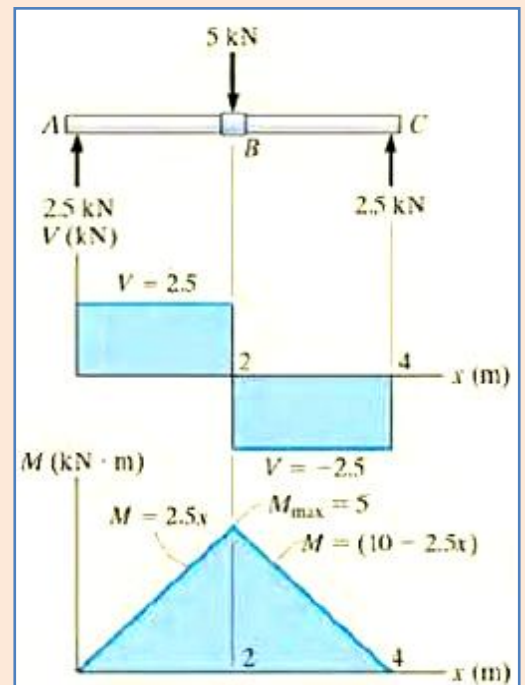
**Exercise 7.5:**

Draw the shear and moment diagrams for the shaft shown in Fig.7-10. The support at A is a thrust bearing and the support at C is a journal bearing.

Fig. 7-10

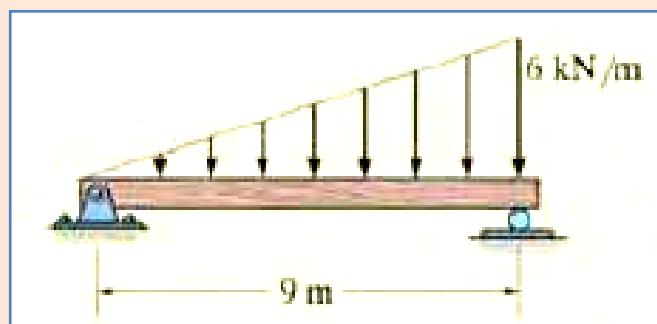


Ans:

**Exercise 7.6:**

Draw the shear and moment diagrams for the beam shown in Fig. 7-11.

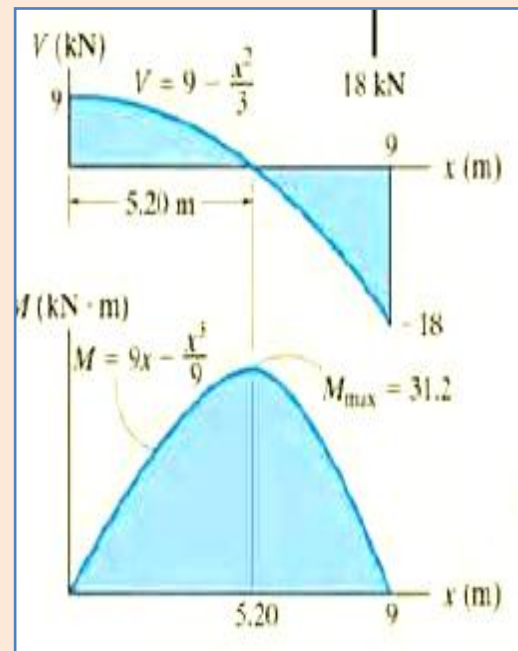
Fig. 7-11



Dr

Abd  
A

Ans:



# Chapter

## Eight

# Friction

اعلم أخي الطالب أن:

النجاح قطرات من المعاناة و الغصص و الجراحات و الآهات و  
المزعجات  
و الفشل قطرات من الخمول و الكسل و العجز و المهانة و  
الخور

## Friction

### 8.1 Characteristics of Dry Friction:

Friction is a **force** that **resists** the **movement** of **two contacting surfaces** that **slide relative to one another**. This force always acts **tangent** to the surface at the points of contact and is directed so as to oppose the possible or existing motion between the surfaces.

In this chapter, we will study, the effects of **dry friction**, which is sometimes called **Coulomb friction** since its characteristics were studied extensively by C. A. Coulomb in 1781. **Dry friction occurs** between the contacting surfaces of bodies **when there is no lubricating fluid**.



### Theory of Dry Friction:

The theory of dry friction can be explained by considering the effects caused by pulling horizontally on a block of uniform weight  $\vec{W}$  which is resting on a rough horizontal surface that is **nonrigid or deformable** Fig. 8- 1a. The upper portion of the block, however, can be considered rigid. As shown on the free-body diagram of the block, Fig. 8-1b, the floor exerts an uneven **distribution** of both **normal force**  $\Delta N_a$  and **frictional force**  $\Delta F_a$  along the contacting surface. For equilibrium, the normal forces must act upward to balance the block's weight  $\vec{W}$ , and the frictional forces act to the left to prevent the applied force  $\vec{P}$  from moving the block to the right. Close examination of the contacting surfaces between the floor and block reveals how these frictional and normal forces develop, Fig. 8- 1c. It can be seen that many microscopic irregularities exist between the two surfaces and, as a result, reactive forces  $\overrightarrow{\Delta R_n}$  are developed at each point of contact. As shown, each reactive force contributes both a frictional component  $\overrightarrow{\Delta F_n}$  and a normal component  $\overrightarrow{\Delta N_n}$ .

الاحتكاك هي القوة المقاومة التي تحدث عند تحرك سطحين متلاصقين باتجاهين متعاكسين عندما يكون بينهما قوة ضاغطة تعمل على تلاصقهما معا.

الاحتكاك الجاف (لا يوجد تشحيم ولا تزييت) ينشأ نتيجة وجود نتوءات وفجوات بين الأسطح. تصطدم هذه النتوءات عند تطبيق قوة (سحب) على الجسم المتحرك مما ينتج قوة احتكاكية بين السطحين. يمكن تجزئة القوى على مستوى السطحين إلى قوتين، ضاغطة و عمودية و معاكسة لقوة وزن الجسم و قوة احتكاك معاكسة لقوة السحب وهي مماسة للسطحين.

**Equilibrium:**

The effect of the **distributed** normal and frictional loadings is indicated by their **resultants**  $\vec{N}$  and  $\vec{F}$ . On the free body diagram, Fig. 8-1d. Notice that  $N$  acts distance  $x$  to the right of the line of action of  $\vec{W}$ . Fig. 8-1d. This location, which coincides with the centroid or geometric center of the normal force distribution in Fig. 8-1b, is necessary in order to balance the "tipping effect" caused by  $\vec{P}$ . For example, if  $\vec{P}$  is applied at a height  $h$  from the surface, Fig. 8-1d, then moment equilibrium about point  $O$  is satisfied if  $w x = ph$  or  $x = ph/w$

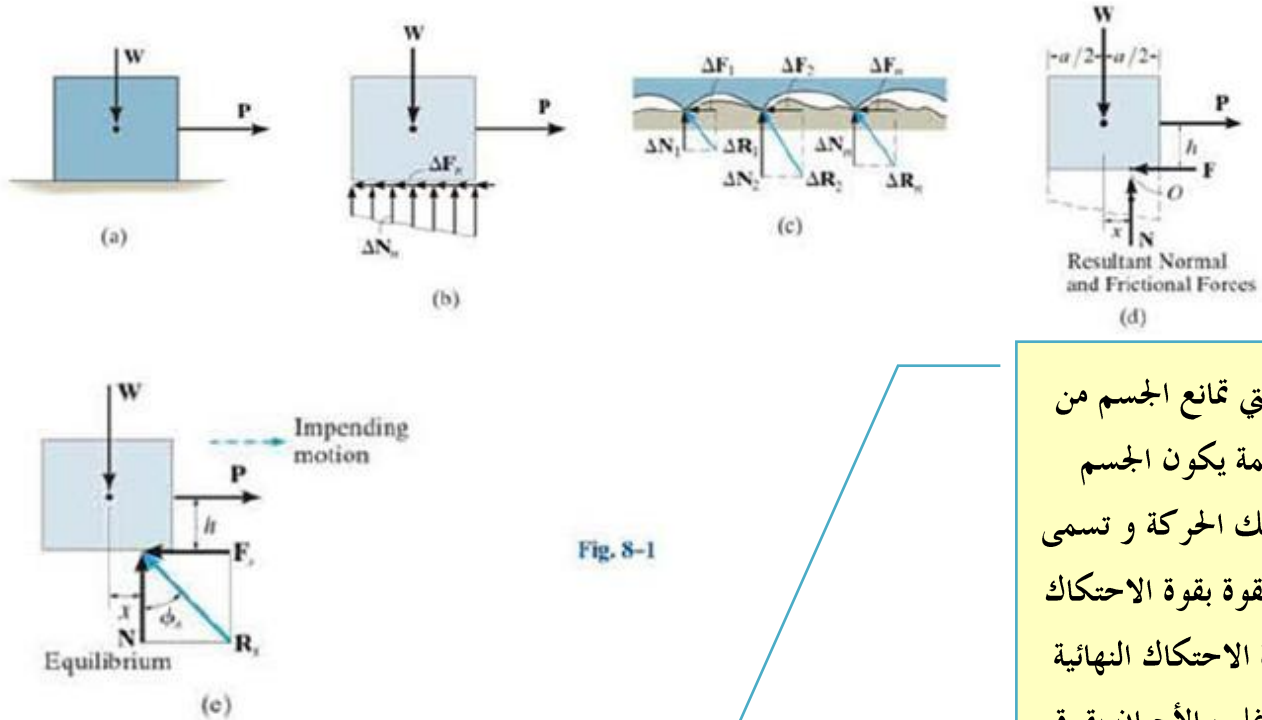


Fig. 8-1

تصل القوة التي تمنع الجسم من الحركة إلى قيمة يكون الجسم فيها على وشك الحركة وتسمى عندئذ هذه القوة بقوة الاحتكاك الحرج أو قوة الاحتكاك النهائية وتعرف في أغلب الأحيان بقوة الاحتكاك الساكن، وهي تتناسب طردياً مع القوة الضاغطة بين الجسمين.

**Impending Motion:**

In cases where the surfaces of contact are rather "slippery", the frictional force  $\vec{F}$  may **not** be great enough to balance  $\vec{p}$ . and consequently the block will tend to slip. In other words, as  $P$  is slowly increased,  $F$  correspondingly increases until it attains a certain **maximum value**  $F_x$  called the limiting static frictional force, Fig. 8-1e. When this value is reached, the block is in **unstable equilibrium** since any further increase in  $P$  will cause the block to move. Experimentally, it has been determined that this **limiting static frictional force**  $F$ , is **directly proportional** to the resultant normal force  $N$ . Expressed mathematically.

$$F_s = \mu_s N$$

معامل الاحتكاك  $\mu_s$  هو كمية عددية تستخدم للتعبير عن النسبة بين قوة الاحتكاك بين جسمين والقوة الضاغطة بينهما، وليس له وحدة قياس، ويعتمد على مادتي الجسمين.



where the constant of proportionality,  $\mu_s$  (mu "sub" s), is called the **coefficient of static friction**.

Thus, when the block is on the **verge of sliding**, the normal force  $\vec{N}$  and frictional force  $\vec{F}_s$  combine to create a resultant  $\vec{R}_s$ , Fig. 8-1e the angle  $\phi_s$  (phi "sub" s) that  $\vec{R}_s$  makes with  $\vec{N}$  is called the **angle of static friction**. From the figure.

$$\phi_s = \tan^{-1} \left( \frac{F_s}{N} \right) = \tan^{-1} \left( \frac{\mu_s N}{N} \right) = \tan^{-1} \mu_s$$

Typical values for  $\mu_s$  are given in Table 8-1. Note that these values can vary since experimental testing was done under variable conditions of roughness and cleanliness of the contacting surfaces. For applications, therefore, it is important that both caution and judgment be exercised when selecting a coefficient of friction for a given set of conditions. When a more accurate calculation of  $F_x$ , is required, the coefficient of friction should be determined directly by an experiment that involves the two materials to be used.

**Table 8-1**  
**Typical Values for  $\mu_s$**

Contact Materials	coefficient of static Friction $\mu_s$
Metal on ice	0.03 - 0.05
Wood on wood	0.30 - 0.70
Leather on wood	0.20 - 0.50
Leather on metal	0.30 - 0.60
Aluminum on aluminum	1.10 - 1.7

### Motion:

If the magnitude of  $\vec{P}$  acting on the block is increased so that it becomes slightly greater than  $F_s$ , the frictional force at the contacting surface will drop to a smaller value  $F_k$ , called the **kinetic frictional force**. The block will begin to slide with increasing speed, Fig. 8-2a. As this Occurs, the block will "ride" on top of these peaks at the points of contact, as shown in Fig. 8-2b. The continued breakdown of the surface is the dominant mechanism creating kinetic friction. Experiments with sliding blocks indicate that the magnitude of the kinetic friction force is directly proportional to the magnitude of the resultant normal force, expressed mathematically as

$$F_k = \mu_k N$$

تتوقف قيمة معامل الاحتكاك على نوع مادة السطحين المتلامسين فهو يختلف من مادة إلى أخرى كما يتوقف على طبيعة السطحين من حيث الخشونة، النعومة، درجة الحرارة، ووجود شوائب والجدول التالي يعطيك قيم معاملات الاحتكاك لبعض المواد وهي في الواقع قيم تقريبية

بمجرد أن يبدأ الجسم في التحرك تقل قوة الاحتكاك عن قيمتها النهائية، وتسمى قوة الاحتكاك هذه، والتي تؤثر في الجسم أثناء حركته بقوة الاحتكاك الحركي. وهي تتناسب طرديا مع القوة الضاغطة بين الجسمين. معامل الاحتكاك الحركي يكون عادة أقل من معامل الاحتكاك الساكن.

Here the constant of proportionality,  $\mu_k$ , is called the **coefficient of kinetic friction**. Typical values for  $\mu_k$  are approximately 25 percent **smaller** than those listed in Table 8-1 for  $\mu_s$ . As shown in Fig. 8-2a, in this case, the resultant force at the surface of contact,  $\vec{R}_k$ , has a line of action defined by  $\phi_k$ . This angle is referred to as the **angle of kinetic friction**, where

$$\phi_s = \tan^{-1} \left( \frac{F_k}{N} \right) = \tan^{-1} \left( \frac{\mu_k N}{N} \right) = \tan^{-1} \mu_k$$

By comparison,  $\phi_s \geq \phi_k$ .

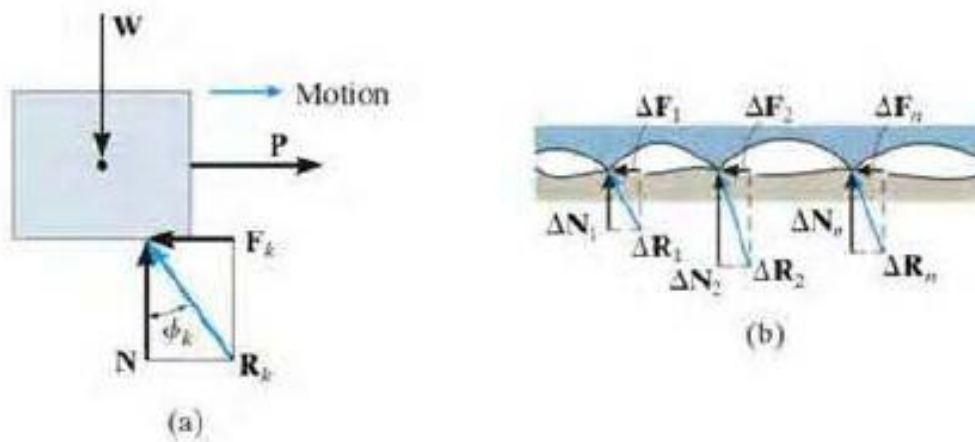


Fig. 8-2

**Exercise 8.1:**

The uniform crate shown in Fig. 8-3 has a mass of 20 kg. If a force  $P = 80$  N is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is  $\mu_s = 0.3$ .

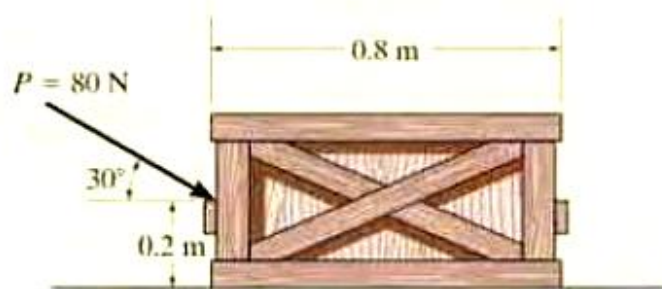
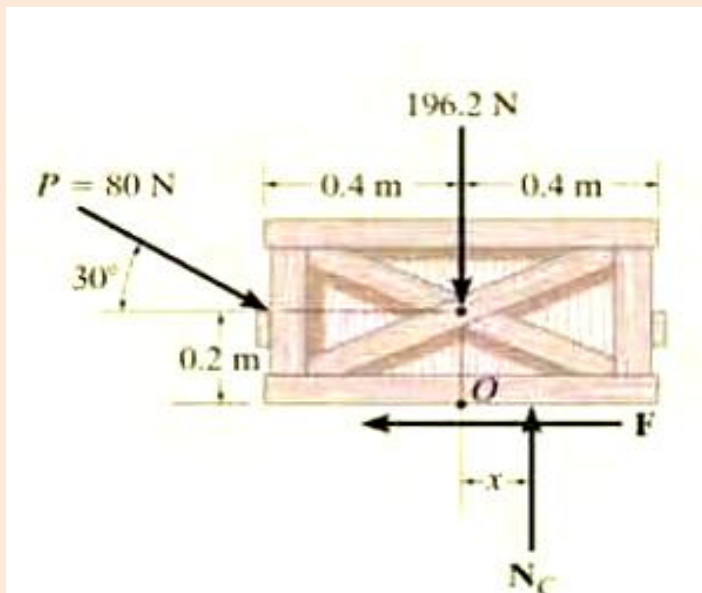


Fig. 8-3

Ans:

Free-Body Diagram:

As shown in Fig. 8-3, the resultant normal force  $\vec{N}_c$  must act a distance  $x$  from the crate's center line in order to counteract the tipping effect caused by  $\vec{P}$ . There are three unknowns  $F$ ,  $N_c$ , and  $x$ , which can be determined strictly from the three equations of equilibrium.



equations of equilibrium:

$$\rightarrow \sum F_x = 0 \quad 80 \cos 30^\circ \text{ N} (0.2\text{m}) + N_c(x) = 0$$

$$+\uparrow \sum F_y = 0 \quad -80 \sin 30^\circ \text{ N} + N_c - 196.2 \text{ N} = 0$$

$$\curvearrowright + \sum M_O = 0 \quad 80 \sin 30^\circ \text{ N}(0.4\text{m}) - 80 \cos 30^\circ \text{ N}(0.2\text{m}) + N_c(x) = 0$$

$$\mathbf{F = 69.3\text{ N}}$$

$$\mathbf{N_c = 236\text{ N}}$$

$$\mathbf{x = -0.00908\text{ m} = -9.08\text{ mm}}$$

Since  $x$  is negative it indicates the resultant normal force acts (slightly) to the left of the crate's center line. No tipping will occur since  $x < 0.4\text{ m}$ . Also, the maximum frictional force which can be developed at the surface of contact is  $F_{\max} = \mu_s N_c = 0.3(236\text{ N}) = 70.8\text{ N}$ . Since  $F = 69.3\text{ N} < 70.8\text{ N}$ , the crate will not slip although it is very close to doing so.

**Exercise 8.2:**

It is observed that when the bed of the dump truck is raised to an angle of  $\theta = 25^\circ$  the vending machines will begin to slide off the bed, Fig. 8-4,. Determine the static coefficient of friction between a vending machine and the surface of the truck bed.

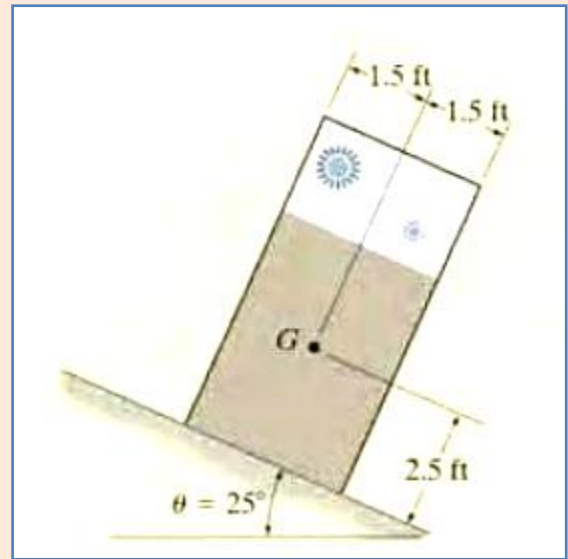


Fig. 8-4

Ans:  $\mu_s = \tan 25^\circ = 0.466$

# Chapter

Nine

## Center Of Gravity And Centroid

سهرى لتتقيح العلوم الذلى  
و تمايلي طربا لحل عويصة  
من وصل غانية و طيب عناق  
أشهى و أحلى من مدامة ساقى

## Center of Gravity and Centroid

### 9.1 Center of Gravity:

A body is composed of an infinite number of particles of differential size, and so if the body is located within a gravitational field, then each of these particles will have a weight  $dW$ , Fig. 9-1a. These weights will form an approximately parallel force system, and the resultant of this system is the total weight of the body, which passes through a single point called the center of gravity,  $G$ , Fig. 9-1b.

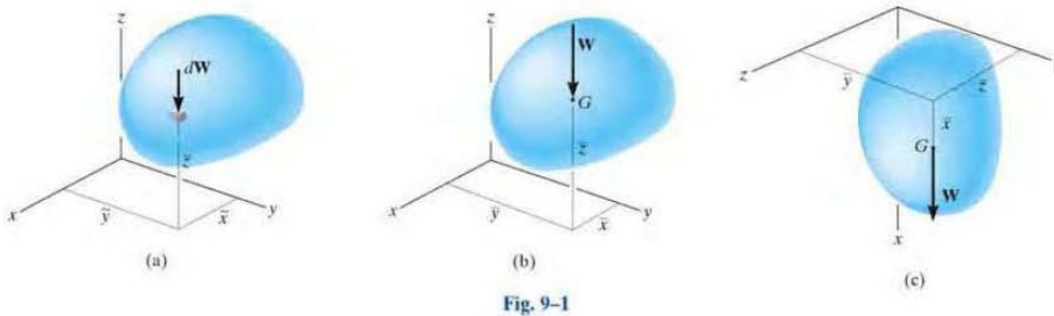


Fig. 9-1

the location of the center of gravity  $G$  with respect to the  $x, y, z$  axes becomes

$$\bar{x} = \frac{\int \tilde{x} dW}{\int dW} \quad \bar{y} = \frac{\int \tilde{y} dW}{\int dW} \quad \bar{z} = \frac{\int \tilde{z} dW}{\int dW}$$

Here  $\bar{x}, \bar{y}, \bar{z}$  are the coordinates of the center of gravity  $G$ , Fig. 9-1b.

$\tilde{x}, \tilde{y}, \tilde{z}$  are the coordinates of each particle in the body, Fig. 9- 1a.

Expressed also as

$$X_{cg} = \frac{X_1 W_1 + X_2 W_2 + X_3 W_3 + \dots}{W_1 + W_2 + W_3 + \dots} = \frac{\sum_1^N X_i W_i}{\sum_1^N W_i}$$

$$Y_{cg} = \frac{Y_1 W_1 + Y_2 W_2 + Y_3 W_3 + \dots}{W_1 + W_2 + W_3 + \dots} = \frac{\sum_1^N Y_i W_i}{\sum_1^N W_i}$$

$$Z_{cg} = \frac{Z_1 W_1 + Z_2 W_2 + Z_3 W_3 + \dots}{W_1 + W_2 + W_3 + \dots} = \frac{\sum_1^N Z_i W_i}{\sum_1^N W_i}$$

يمكن تصور تأثير الجاذبية الأرضية على أي جسم على أنه ناتج تأثيراتها على الجسيمات المكونة لهذا الجسم.

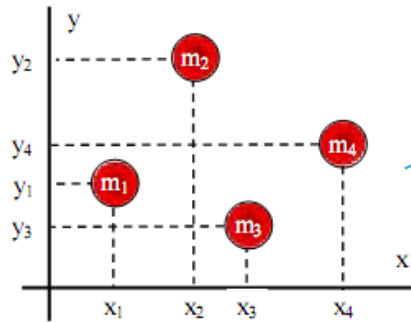
فبتجميع كل القوى المؤثرة على الجسيمات المكونة للجسم نحصل على قوة جاذبي الكرة الأرضية للجسم و هو ما نسميه قوة وزن الجسم، لذا يعرف مركز ثقل الجسم على أنه النقطة من الفراغ التي يمر بها دائما خط عمل قوة وزن هذا الجسم، وهي نقطة وحيدة و ثابتة بالنسبة لهذا الجسم.

لو فرضنا بأن لدينا نظاما ما يحتوي على عدد من الأجسام وزنها  $W_1, W_2, \dots, W_3$  موزعة في المستوي  $x, y, z$ ، فمركز الثقل لهذا النظام يقع في النقطة التي إحداثياتها  $(X_{cg}, Y_{cg}, Z_{cg})$ .

**9.2 Center of Mass of a Body:**

In order to study the dynamic response or accelerated motion of a body, it becomes important to locate the body's center of mass  $C_m$ , Fig. 9-2. This location can be determined by substituting  $dW = g \, dm$  into last equations. Since  $g$  is constant, it cancels out, and so

$$\bar{x} = \frac{\int \tilde{x} \, dm}{\int dm} \quad \bar{y} = \frac{\int \tilde{y} \, dm}{\int dm} \quad \bar{z} = \frac{\int \tilde{z} \, dm}{\int dm}$$



مركز الكتلة هو النقطة التي تؤثر فيها محصلة الكتل، فلو فرضنا بأن نظاما ما يحتوي على عدد من الأجسام كتلة كل منهما  $m_1$ ،  $m_2$ ،  $m_3$ .... موزعة في المستوي  $x$ ،  $y$ ،  $z$ ، فمركز الكتلة لهذا النظام يقع في النقطة التي إحداثياتها  $(Z_{cm}, Y_{cm}, X_{cm})$ .

Expressed also as

$$X_{cm} = \frac{m_1 X_1 + m_2 X_2 + m_3 X_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_1^N m_i X_i}{\sum_1^N m_i}$$

$$Y_{cm} = \frac{m_1 Y_1 + m_2 Y_2 + m_3 Y_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_1^N m_i Y_i}{\sum_1^N m_i}$$

$$Z_{cm} = \frac{m_1 Z_1 + m_2 Z_2 + m_3 Z_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_1^N m_i Z_i}{\sum_1^N m_i}$$

**9.3 Centroid of a Volume:**

If the body in Fig. 9-3 is made from a homogeneous material, then its density  $\rho$  (rho) will be constant. Therefore, a differential element of volume  $dV$  has a mass  $dm = \rho \, dV$ . Substituting this into the next Equations and canceling out  $\rho$ , we obtain formulas that locate the centroid  $C$  or geometric center of the body, namely

$$\bar{x} = \frac{\int \tilde{x} \, dV}{\int dV} \quad \bar{y} = \frac{\int \tilde{y} \, dV}{\int dV} \quad \bar{z} = \frac{\int \tilde{z} \, dV}{\int dV}$$



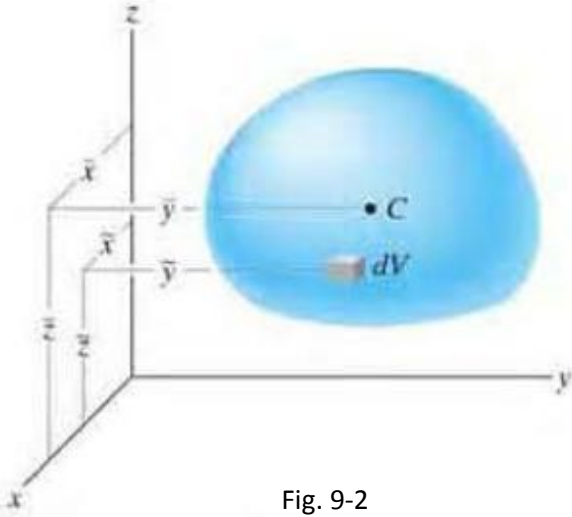


Fig. 9-2

We can express the centroid of a volume as

$$X_{cv} = \frac{X_1 V_1 + X_2 V_2 + X_3 V_3 + \dots}{V_1 + V_2 + V_3 + \dots} = \frac{\sum_1^N X_i V_i}{\sum_1^N V_i}$$

$$Y_{cv} = \frac{Y_1 V_1 + Y_2 V_2 + Y_3 V_3 + \dots}{V_1 + V_2 + V_3 + \dots} = \frac{\sum_1^N Y_i V_i}{\sum_1^N V_i}$$

$$Z_{cv} = \frac{Z_1 V_1 + Z_2 V_2 + Z_3 V_3 + \dots}{V_1 + V_2 + V_3 + \dots} = \frac{\sum_1^N Z_i V_i}{\sum_1^N V_i}$$

لو فرضنا بأن لدينا نظاما ما  
يحتوي على عدد من  
الأجسام حجمها  $V_1, V_2, \dots, V_3$   
موزعة في المستوي  
 $x, y, z$  ، فمركز موضع  
الحجم لهذا النظام يقع في  
النقطة التي إحداثياتها  
( $X_{cv}, Y_{cv}, Z_{cv}$ )

## 9.4 Centroid of an Area:

If an area lies in the x-y plane and is bounded by the curve  $y = f(x)$ , as shown in Fig. 9-3a, then its centroid will be in this plane and can be determined from integrals similar to the next equations, namely,

$$\bar{x} = \frac{\int_A \tilde{x} dA}{\int_A dA}$$

$$\bar{y} = \frac{\int_A \tilde{y} dA}{\int_A dA}$$

We can express the centroid of an area as

$$X_{cA} = \frac{X_1 A_1 + X_2 A_2 + X_3 A_3 + \dots}{A_1 + A_2 + A_3 + \dots} = \frac{\sum_1^N X_i A_i}{\sum_1^N A_i}$$

$$Y_{cA} = \frac{Y_1 A_1 + Y_2 A_2 + Y_3 A_3 + \dots}{A_1 + A_2 + A_3 + \dots} = \frac{\sum_1^N Y_i A_i}{\sum_1^N A_i}$$

مركز موضع المساحة

لو فرضنا بأن لدينا نظاما ما  
يحتوي على عدد من  
الأجسام مساحتها  $A_1, A_2, \dots, A_3$   
موزعة في المستوي  
 $x, y$  ، فمركز موضع  
المساحة لهذا النظام يقع في  
النقطة التي إحداثياتها  
( $X_{cA}, Y_{cA}$ )

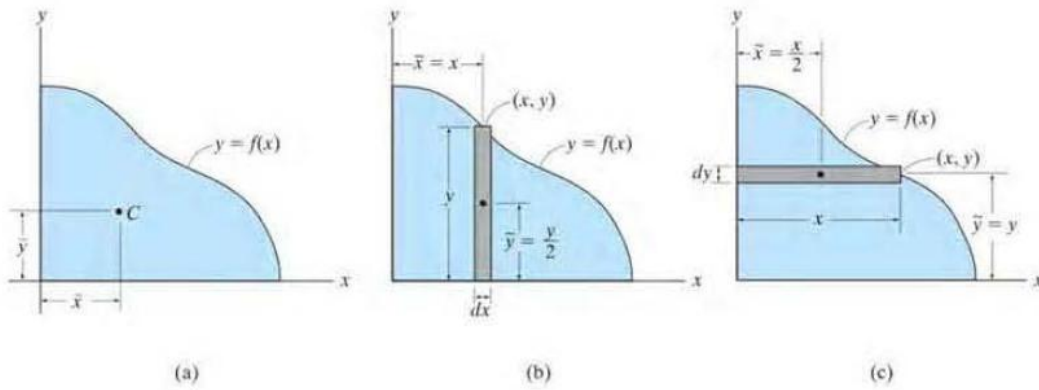


Fig. 9-3

### 9.5 Centroid of a Line:

مركز موضع الخط

If a line segment (or rod) lies within the x-y plane and it can be described by a thin curve  $y = f(x)$ , Fig. 9-4a, then its centroid is determined from

$$\bar{x} = \frac{\int_L \tilde{x} dL}{\int_L dL} \qquad \bar{y} = \frac{\int_L \tilde{y} dL}{\int_L dL}$$

Here the length of the differential element is given by the Pythagorean theorem,  $dL = \sqrt{(dx)^2 + (dy)^2}$ , which can also be written in the form

$$\begin{aligned} dL &= \sqrt{\left(\frac{dx}{dx}\right)^2 dx^2 + \left(\frac{dy}{dx}\right)^2 dx^2} \\ &= \left( \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \right) dx \end{aligned}$$

or

$$\begin{aligned} dL &= \sqrt{\left(\frac{dx}{dy}\right)^2 dy^2 + \left(\frac{dy}{dy}\right)^2 dy^2} \\ &= \left( \sqrt{\left(\frac{dx}{dy}\right)^2 + 1} \right) dy \end{aligned}$$

Either one of these expressions can be used: however, for application, the one that will result in a simpler integration should be selected. For example, consider the rod in Fig. 9-4b, defined by  $y = 2x^2$ . The length of

The element is  $dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ , and since  $\frac{dy}{dx} = 4x$ , then  $dL = \sqrt{1 + (4x)^2} dx$ . The centroid for this element is located at  $\tilde{x} = x$  and  $\tilde{y} = y$ .

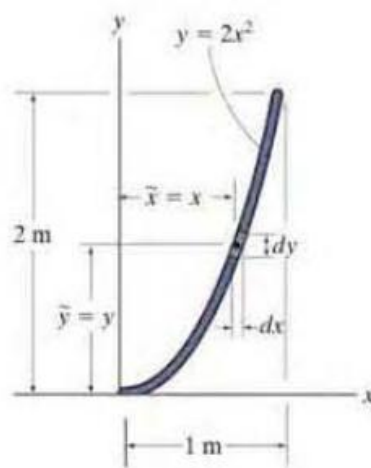
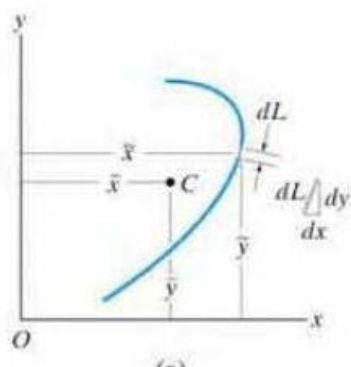
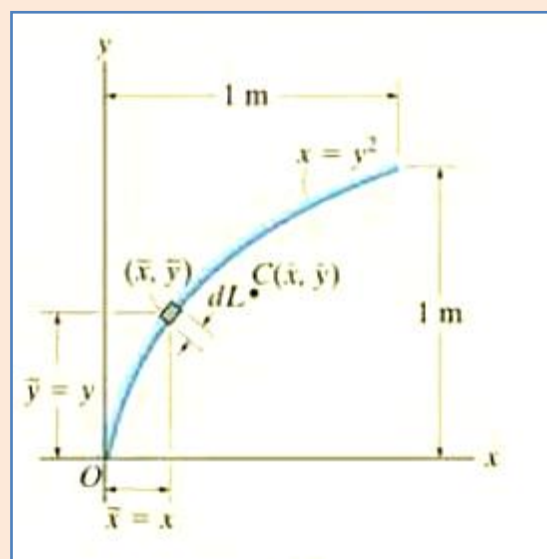


Fig. 9-4

**Exercise 9.1:**

Locate the centroid of the rod bent into the shape of a parabolic arc as shown in Fig. 9-5.

Fig. 9-5



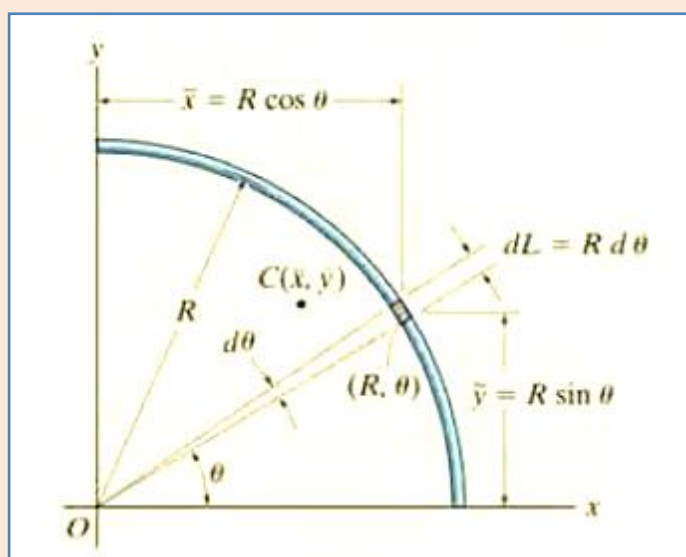
Ans:  $\bar{x} = 0.410 \text{ m}$

$\bar{y} = 0.574 \text{ m}$

**Exercise 9.2:**

Locate the centroid of the circular wire segment shown in Fig. 9-6.

Fig. 9-6



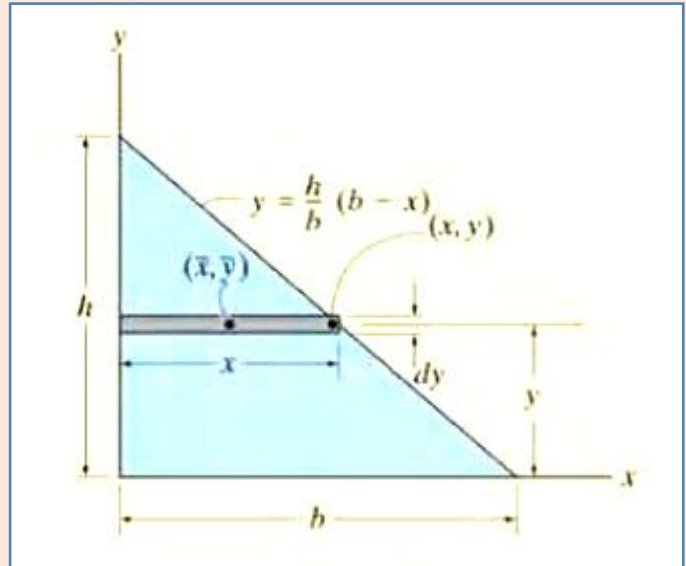
Ans:  $\bar{x} = \frac{2R}{\pi}$

$\bar{y} = \frac{2R}{\pi}$

**Exercise 9.3:**

Determine the distance  $\bar{y}$  measured from the  $x$  axis to the centroid of the area or the triangle shown in Fig. 9-7.

Fig. 9-7



Ans:  $\bar{y} = \frac{h}{3}$

# Chapter

Ten

## Moments Of Inertia

بعض فوائد القراءة و المطالعة:

- ✓ اجتناب الخوض في الباطل.
- ✓ تنمية العقل ، وتجويد الذهن ، وتصفية الخاطر .
- ✓ غزارة العلم ، وكثرة الحفوظ والمفهوم .
- ✓ راحة للذهن من التششت ، وللقلب من التشرذم ، وللوقت من الضياع .

## Moments of Inertia

### 10.1 Moment of Inertia:

By definition, the moments of inertia of a differential area  $dA$  about the  $x$  and  $y$  axes are  $dl_x = y^2 dA$  and  $dl_y = x^2 dA$ , respectively, Fig.10-1. For the entire area  $A$  the moments of inertia are determined by integration.

عزم القصور الذاتي هو مصطلح فيزيائي يعبر عن مدى ميل الجسم الساكن لمقاومة التأثيرات الخارجية المفروضة عليه.

فمثلا عندما يتعرض الجسم إلى قوى و اجهادات فان الذي يقاومها هو سطح المقطع العرضي و بالتالي فان تغيير شكل و أبعاد المقطع العرضي له تأثير كبير على سلوك المادة تحت تأثيرات الاجهادات. لذلك يعتمد حساب عزم القصور الذاتي على الأبعاد الهندسية و نجد بأن له أهمية كبيرة في مسائل التي تتعلق بانحناء الكمرات.

$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$

معادلة عزم القصور الذاتي لعنصر مساحي  $dA$  بالنسبة لأي محور هو حاصل ضرب العنصر المساحي مع مربع المسافة بين مركزه و المحور.

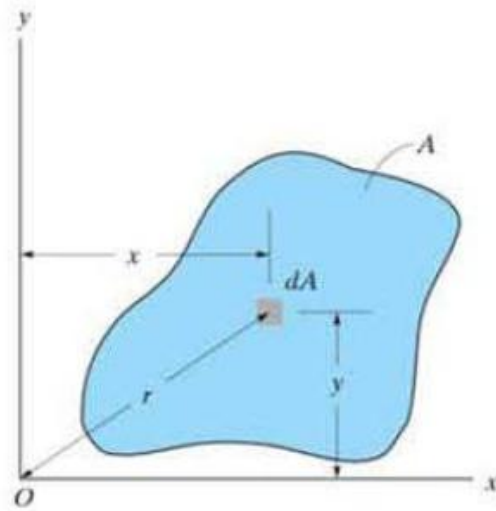


Fig. 10-1

We can also formulate this quantity for  $dA$  about the "pole"  $O$  or  $z$  axis, Fig. 10-1. This is referred to as the **polar moment of inertia**. It is defined as  $dJ_o = r^2 dA$  where,  $r$  is the perpendicular distance from the pole ( $z$  axis) to the element  $dA$ . For the entire area the polar moment of inertia is

عزم القصور الذاتي القطبي

$$J_o = \int_A r^2 dA = I_x + I_y$$

This relation between  $J_O$  and  $I_x, I_y$  is possible since  $r^2 = x^2 + y^2$ , Fig. 10-1. From the above formulations it is seen that  $I_x, I_y$  and  $J_O$  will always be **positive** since they involve the product of distance squared and area. Furthermore, the units for moment of inertia involve length raised to the fourth power, e. g.  $m^4, mm^4, \text{ or } ft^4, in^4$

## 10.2 Parallel-Axis Theorem for an Area

The **parallel-axis theorem** can be used to find the moment of inertia of an area about **any** axis that is parallel to an axis passing through the centroid and about which the moment of inertia is known. To develop this theorem, we will consider finding the moment of inertia of the shaded area shown in Fig. 10-2 about the axis. To start, we choose a differential element  $dA$  located at an arbitrary distance  $y'$  from the **centroidal**  $x'$  axis. If the distance between the parallel  $x'$  and  $x$  axes is  $d_y$ , then the moment of inertia of  $dA$  about the  $x$  axis is  $dl_x = (y' + d_y)^2 dA$ . For the entire area.

$$I_x = \int_A (y' + d_y)^2 dA = \int_A y'^2 dA + d_y \int_A y' dA + d_y^2 \int_A dA$$

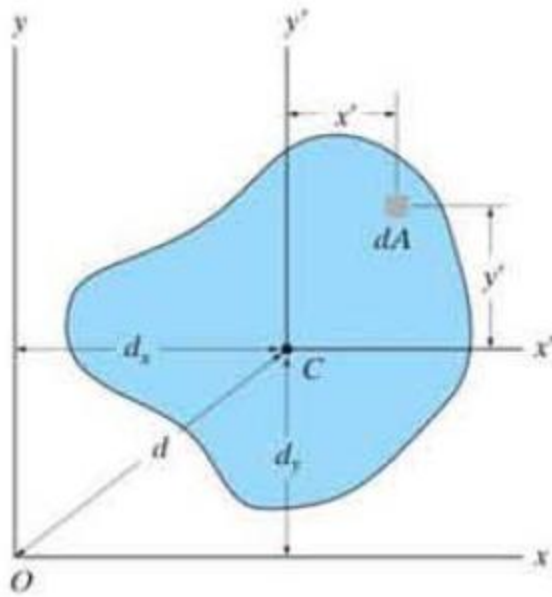


Fig. 10-2

The first integral represents the moment of inertia of the area about the centroidal axis,  $I_{x'}$ . The second integral is zero since the  $x'$  axis passes through the area's centroid  $C$ ; i.e.,  $\int y' dA = \bar{y} \int dA = 0$  since  $\bar{y} = 0$ . Since the third integral represents the total area  $A$ , the final result is Therefore

$$I_x = \bar{I}_{x'} + Ad_y^2$$

A similar expression can be written for  $I_y$ , i. e.,

$$I_y = \bar{I}_{y'} + Ad_x^2$$

حساب عزم القصور الذاتي  
للمساحة بناء على نظرية  
المحاور المتوازية يتطلب  
إدراج محورين موازيين  
للمحاور الأساسية و  
يتقاطعان في موضع مركز  
المساحة.



And finally, for the polar moment of inertia, since

$$\bar{J}_C = \bar{I}_{x'} + \bar{I}_{y'}$$

And

$$d^2 = d_x^2 + d_y^2$$

we have

$$J_O = \bar{J}_C + Ad^2$$

The form of each of these three equations states that **the moment of inertia for an area about an axis is equal to its moment of inertia about a parallel axis passing through the area's centroid plus the product of the area and the square of the perpendicular distance between the axes.**

### 10.3 Radius of Gyration of an Area

The **radius of gyration** of an area about an axis has units of length and is a quantity that is often used for the design of columns in structural mechanics. Provided the areas and moments of inertia are **known**, the radii of gyration are determined from the formulas.

$$k_x = \sqrt{\frac{I_x}{A}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

$$k_O = \sqrt{\frac{J_O}{A}}$$

يعرف نصف قطر القصور  
لمساحة بالنسبة لمحور  
معين بأنه الطول الذي إذا  
ربع و ضرب بالمساحة  
يعطي عزم القصور الذاتي  
بالنسبة للمحور المذكور.

لنصف القطر القصور  
الذاتي أهمية خاصة في  
المسائل التي تتعلق بانبعاج  
الأعمدة.

the form of these equations is easily remembered since it is similar to that for finding the moment of inertia for it differential area about an axis. For example,  $I_x = K_x^2 A$  whereas for a differential area,  $dI_x = y^2 dA$ .

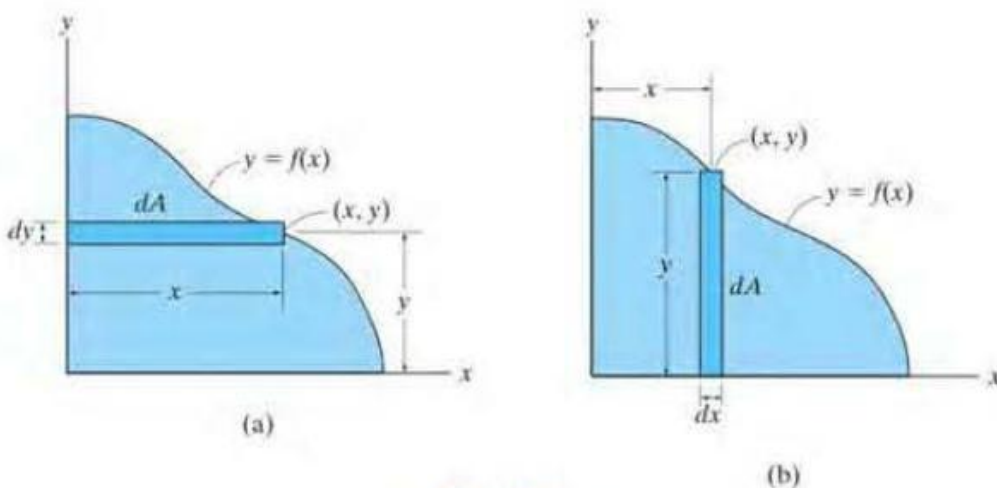


Fig. 10-3

**Exercise 10.1:**

Determine the moment of inertia for the rectangular area shown in Fig. 10-4 with respect to (a) the centroidal,  $x'$  axis, (b) the axis,  $x_b$  passing through the base of the rectangle, and (c) the pole or  $y'$  axis perpendicular to the  $x' - y'$  plane and passing through the centroid C.

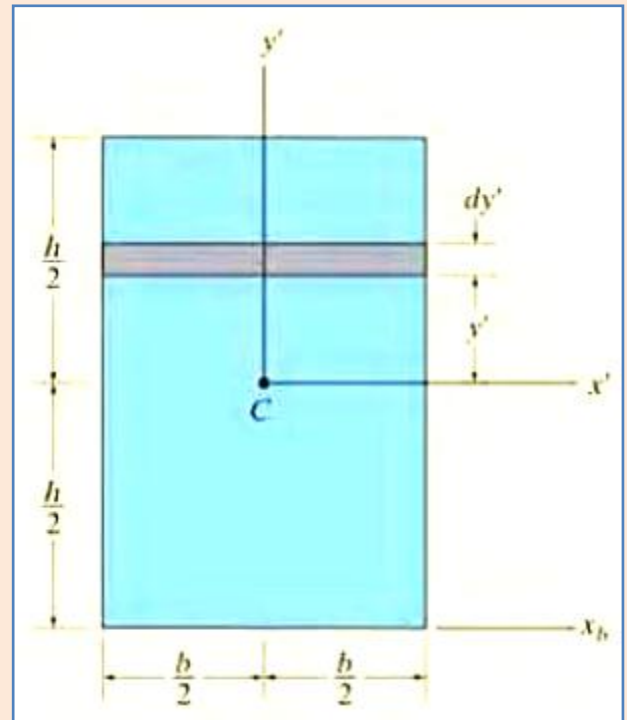


Fig. 10-4

Ans:  $I_x = \frac{1}{12}bh^3$        $I_{x_b} = I_x + Ad_y^2 = \frac{1}{3}bh^3$        $I_y = \frac{1}{12}hb^3$        $J_C = I_x + I_y = \frac{1}{12}bh(h^2 + b^2)$

**Exercise 10.2:**

Determine the moment of inertia for the shaded area shown in Fig. 10-5 about the x axis.

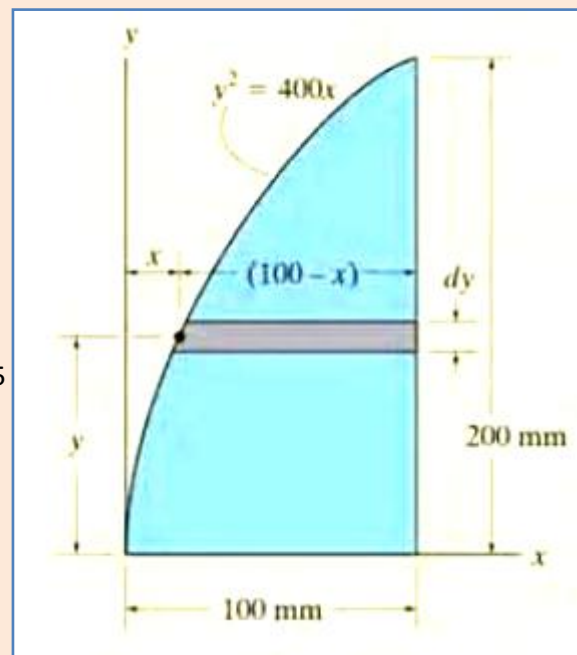


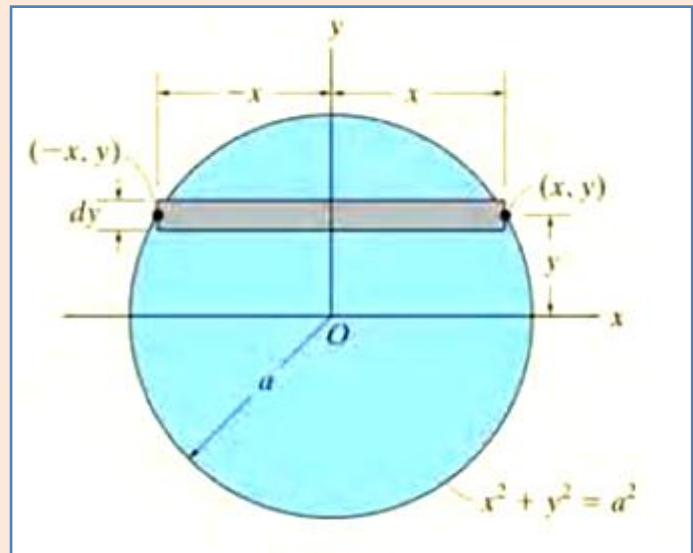
Fig. 10-5

Ans:  $I_x = 107(10^6) \text{ mm}^4$

**Exercise 10.3:**

Determine the moment of inertia with respect to the x axis for the circular area shown in Fig. 10-6.

Fig. 10-6



Ans:  $I_x = \frac{\pi a^4}{4}$

انتهت بحمد الله  
مذكرة الإستاتيكا  
لطلاب كلية الهندسة

لا تنسونا من صالح دعائكم مع تحيات د. عبدالوهاب